

Investigation of the resonant properties of wires as a base for complex materials construction

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Abstract

In this paper we consider the problem of diffraction of electromagnetic wave on the resonance wire structure. The resonance properties of this systems gives ability to create the periodical structure of these wires embedded in dielectric in order to get the complex material with desired properties. We consider the mathematical approach of this problem and also the computer simulation of the system of wire pairs radiated by the plane wave.

I. Mathematical approach

First we consider the case of one wire. The problem is the diffraction of the electromagnetic waves on the metal wire of the arbitrary shape and finite length. Suppose the electromagnetic wave harmonic in the time propagating in the homogenous dielectric ε , μ falls on the metal wire l of arbitrary form and finite length. We have to find the field scattered by the wire \vec{E}^s , \vec{H}^s . σ_s and \vec{J}_s are the densities of the charge and current on the surface S of the wire. The scattered field must satisfy the Maxwell equations and the boundary conditions: the total field tangent component must be zero through the S

$$\vec{n} \times (\vec{E}^i + \vec{E}^s) = 0.$$

The vectors \vec{E}^s and \vec{H}^s can be represented by the scalar and vector potentials

$$\vec{E}^s = -j\omega\vec{A} - \vec{\nabla}\varphi, \quad \vec{H}^s = \frac{1}{\mu}\vec{\nabla} \times \vec{A},$$

they satisfies the equations:

$$\nabla^2 \vec{A} + k^2 \vec{A} = -\vec{J}_s \mu, \quad \nabla^2 \varphi + k^2 \varphi = -\frac{\sigma_s}{\varepsilon},$$

$k^2 = \omega^2 \mu \varepsilon$ and also

$$\sigma_s = -\frac{1}{j\omega} \vec{\nabla} \cdot \vec{J}_s.$$

If we suppose that the wire is very thin we can say that:

$$\vec{H}^s = \frac{1}{\mu} \vec{\nabla} \times \vec{A}, \quad \varphi = \frac{1}{4\pi\varepsilon} \int_l \sigma(l) \frac{e^{-jkr}}{r} dl, \quad \vec{A} = \frac{\mu}{4\pi} \int_l \vec{I}(l) \frac{e^{-jkr}}{r} dl, \quad \sigma = -\frac{1}{j\omega} \frac{dI}{dl}$$

where $\sigma(l)$ and $I(l)$ - are the linear densities of the charge and current in the wire. The boundary conditions can be represented as:

$$-E_l^i = -j\omega A_l - \frac{\partial \varphi}{\partial l}.$$

With the method considered in the [1], the wire is divided with the points into elementary segments across which the charge and current densities are considered as constant values.

Each n -th segment is determined by the central point n and the edge points \bar{n} and \bar{n}^+ .

Δl_n^- is the distance between \bar{n} and \bar{n}^+ points. Δl_n^+ and Δl_n^- are the same distances shifted

with half of interval respectively right or left side. The integrals are hanged with summation on these segments and the derivatives – with finite differences on the same segments

$$-E^i(m) \approx -j\omega A(m) - \frac{\varphi\left(m^+\right) - \varphi\left(m^-\right)}{\Delta l_m}, \quad (1) \quad \varphi\left(m^+\right) \approx \frac{1}{4\pi\epsilon} \sum_n \sigma\left(n^+\right) \int_{\Delta l_n^+} \frac{e^{-jkr}}{r} dl, \quad (3)$$

$$\bar{A}(m) \approx \frac{\mu}{4\pi} \sum_n \bar{I}(n) \int_{\Delta l_n} \frac{e^{-jkr}}{r} dl, \quad (2) \quad \sigma\left(n^+\right) \approx -\frac{1}{j\omega} \left[\frac{I(n+1) - I(n)}{\Delta l_n^+} \right]. \quad (4)$$

In order to determine the current across the segments, we can make the matrix equation $[V] = [Z][I]$,

where $[V]$, $[Z]$ and $[I]$ - are the matrixes of voltage, impedance and current respectively.

The elements of voltage matrix have the form $\bar{E}^i \cdot \Delta \bar{l}_n$.

The elements of impedance matrix are the next expression.

$$Z_{nm} = j\omega\mu\Delta l_n\Delta l_m\psi(n,m) + \frac{1}{j\omega\epsilon} \left[\psi\left(n^+, m^+\right) - \psi\left(n^-, m^+\right) - \psi\left(n^+, m^-\right) + \psi\left(n^-, m^-\right) \right],$$

$$\psi(n,m) = \frac{1}{\Delta l_n \Delta l_m} \int \frac{e^{-jkr_{nm}}}{4\pi r_{nm}} dl_n.$$

After determination the current matrix, form (4) we can find the charge densities on the each elementary segments, and also the scalar and vector potentials form the equations (2) and (3). The scattered field we can find with (1).

II. Computer simulation

Using the mathematical method described above we consider the problem of the plane wave diffraction on the wire pair showed on the Fig.1. It is the circular wires which have open sector with angle 30 degree and located in YOZ plane. The first wire is oriented so that the open sector is in the opposite side to the second wire's open sector. The relation of the wire radiuses is - 2/1.

Incident field is a plane wave propagating in the direction of X axis with Z polarization.

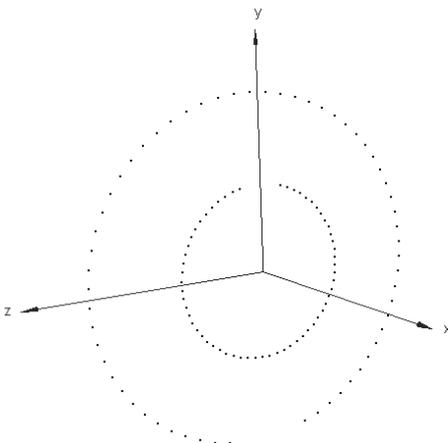


Fig.1
Wire pair geometry

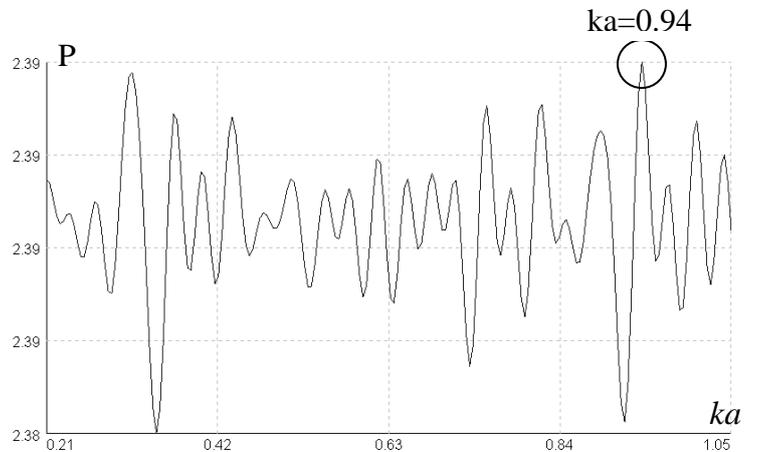


Fig.2
Reflected power dependence on the ka

Reflected power is calculated by summation the pointing vector on the sphere with radius two orders bigger than the linear dimension of the wire pair. This is the energy the system of the wire takes from incident wave and then reradiates – in other words, resonant characteristic (Fig.2).

As it could be seen, this structure has complex characteristic (Fig. 2). This is reflected power dependence on the wave number. It consists of the big amount of minimums and maximums but at the certain frequency it has global maximum. This is the resonant characteristic of this system.

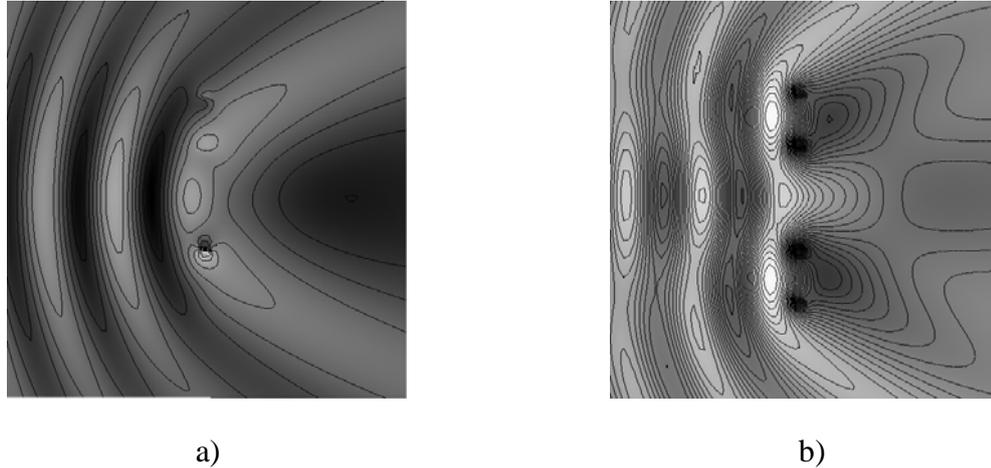


Fig.3
Near field distribution at the resonance frequency
a) YOX section b) ZOX section

Near field distribution at the highest resonance (Fig.2) for two different sections are represented on the Fig. 3 a), b). Fig.4 represents far field pattern radiated by that wire pair. As we see the system has high front and back radiated pattern.

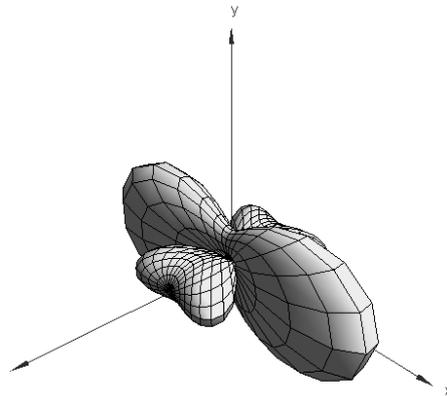


Fig.4
Far zone pattern

We considered also the system of such resonant wire pairs. Our final goal is to investigate the periodical system behavior located into the dielectric because such system has to have the properties of complex materials - anisotropic, bianisotropic, and chiral media. As a first step we would like to study finite periodical structure like on Fig 5.

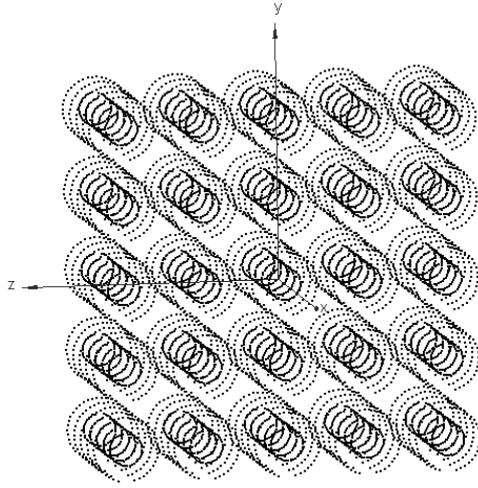


Fig. 5
Wire periodical system geometry

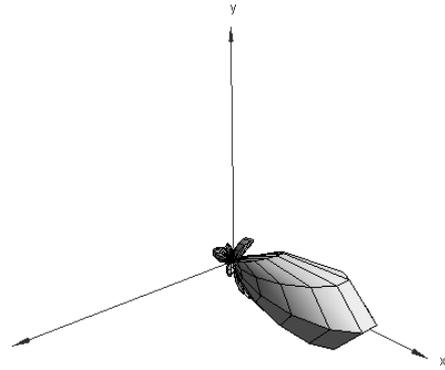
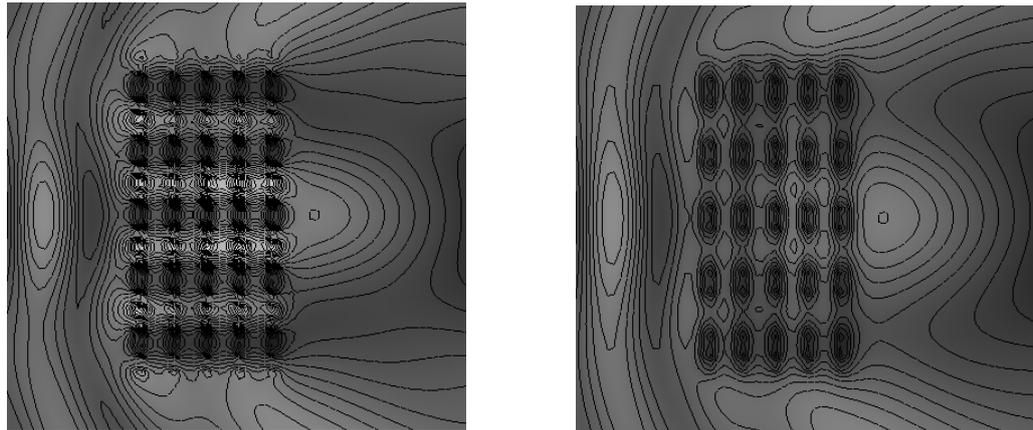


Fig. 6
Far field pattern

We considered the case of the above given wire's systems periodical structure with 5X5X5 elements (Fig.5). The relation between periods of this lattice by X, Y and Z is 1.3X2.5X2.5. Such amount is necessary to obtain the properties of metamaterials. Fig. 6 represents far field pattern for this structure at the same frequency.

The near field distribution in different sections is represented on the Fig. 7



a)

Fig. 7

b)

Near field distribution at the
resonance frequency
a) YOX section b) ZOX section

Conclusion

In this article was considered the diffraction of plane electromagnetic waves on the resonance wire pairs and for wire system. Are obtained some results.

In the future we plan to investigate the periodical system of this wire pair system embedded in the dielectric. It has to have the properties of complex materials such as chiral, anisotropic and bianisotropic media. The new results will be presented at the conference.

References:

[1]. Roger F. Harrington, Donald R. Wilton, Chalmers M. Butler, Raj Mittra, C. Leonard Bennet, Lectures on computational methods in electromagnetics. 1981