

THE METHOD OF AUXILIARY SOURCES IN APPLIED ELECTRODYNAMICS

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INTRODUCTION. The basic problem arising in applied electrodynamics is to calculate EM fields of complex systems and devices. For the efficient utilization of these systems in practice, it is necessary to investigate them previously theoretically and determine their physical properties as full as possible. For this purpose one must use a computation experiment, which consists of a construction of a mathematical model simulation, which is equivalent to the given electrodynamic system. The constructed model must be simple for calculations and its physical properties must be close to those of the real system. During the run of complex calculation, refinement of the mathematical simulation model and optimization of the parameters of the system according to given standards of quality are carried out. Therefore expensive full-scale experiments are replaced by computer simulations.

Proceeding from this, the development of high-speed multipurpose numerical techniques for solving the problems of complex shape bodies and complicates incident field is relevant and very important. The auxiliary sources' technique was used for numerical experiments and was improved during the solution of some electrodynamics problems using computation experiments.

The main idea - to eliminate singularity in the singular integral equation by shifting the contour of sources relative to contour of integration was used by Kupradze V. In [1,2] was proved the completeness and linear independence of a system of fundamental solutions of the Helmholtz equation when their poles are distributed on a closed contour in a nonphysical region (inside the body). Completeness and linear independence of metaharmonic functions of the particular solutions of the Helmholtz equation for homogeneous medium were proved by Vekua I.N. [3-5].

It was assumed in these works that these conditions (completeness and the linear independence) guarantee that there is a linear combination of elements of the set that converges on the boundary to the prescribed boundary values, in the mean-square sense, when the number of elements increases. Thus these authors have offered an approximate method of auxiliary sources for the solution of boundary problems of mathematical physics. In the first case the required solution - scattered field represented by a sum of elementary sources, in the second case by a sum of multipole elementary sources. From the physical point of view the identity of these methods is evident. The known addition theorem will allow us to prove it mathematically.

There is number of works [9-11], the authors of which, probably independently of [1-5], offered just such representations of fields. Frequently they rested on physical causes. This method was successfully used by many researchers for the solution of a wide group of problems - theory of elasticity and geophysics [14], electrostatic problems, hydrodynamics, diffraction of acoustic and electromagnetic waves [15,18-22,24,25].

During numerical realizations there were cases of bad convergence and even divergence of the solution. These problems were discussed at a symposium [23], and in the works of many authors [9-11,24,25]. Basically these problems are connected with a known Rayleigh hypothesis.

By the middle of the 80-is, the main principles of the construction of the solutions of diffraction problems were formulated at the basis of the method of auxiliary sources (MAS). It was established that the preliminary analysis of the geometry of a scatterer is of crucial importance.

Geometry and the incident field determine the location and character of main singularities of scattered fields, which in turn dictate which auxiliary sources should be used to construct the solution of the initial boundary problem and where they should be disposed?

The next most important step is the process of calculation of values of auxiliary sources. As a rule, this problem is reduced to the solution of some system of linear algebraic equations. In this case the main part of machine resources is spent on the calculation of matrix elements and the solution of this system. It has been shown that the optimum method is a collocation method, where the search of amplitudes of auxiliary sources and their phases is reduced to the determination of the pseudo-solution of some redefined (in a general case) system of linear algebraic equations. Then this system has a minimal size and simple matrix elements.

The modern algorithm of MAS is developed and generalized further as a numerical method of the solution of the boundary problem for a Helmholtz equation. In these articles the required solution is suggested as a series of the fundamental solutions of a Helmholtz equation or as multipole sources with centers of radiation on some auxiliary surface located inside the scatterer.

Some of the authors considered such an approach only as a mathematical way of constructing solutions of the initial physical problem and did not take care to the physical meaning. Similarly, the problems arising in numerical realization, which in some cases did not allow a reasonable approximation of the required solutions, were considered purely mathematical (bad properties of matrix of a system of equations, instability of the solutions, accumulation of errors during the calculation, etc). It has been shown that the basic difficulties arise when all physical properties of the scattered field are not taken to account in the algorithm. For instance, the auxiliary surface does not cover all main singularities, or the auxiliary sources do not describe specifics of given singularities.

A similar situation arises in the scattering problem of electromagnetic waves on edge possessing bodies [28]. Here the required field should have necessary singularities near the edge (Meixners condition).

In the present work, based on the concept of main singularities of the scattered field an algorithm of the MAS is given. This algorithm allows one to perform the calculations in a real time scale with guaranteed accuracy. In the beginning conventional MAS is given, some generalizations are made and on the basis of the analysis of problems arising in this method the concept of main singularities are introduced. Further, the specific character of the construction of the solutions of some applied problems is given on the basis of stated generalization.

1. CONVENTIONAL METHOD OF AUXILIARY SOURCES

Suppose it is necessary to investigate a diffraction problem of linearly polarized, monochromatic electromagnetic wave on a body by surface S (perfectly conductor or with final conductivity; homogeneous, anisotropic dielectric with absorption). The time dependence is as $e^{-i\omega t}$ and incident field is a field of plane wave or field of dipole source (magnetic or electric).

As is known, the given problem is reduced to the solution of the following boundary problem for the Helmholtz equation: to find a solution of the following equation

$$\Delta U^s(x, y, z) + k^2 U^s(x, y, z) = 0, \quad (1)$$

and the validity of the boundary conditions on the surface S :

$$W\{U^s(x, y, z) - U^i(x, y, z)\} = 0, \quad M(x, y, z) \in S, \quad (2)$$

where $U^s(x, y, z)$ and $U^i(x, y, z)$ are the functions describing scattered and incident fields respectively; W an operator describing the boundary conditions.

According to MAS the Auxiliary Sources are distributed on the surface σ , which is selected to be always inside of the non-physical area of the scatterer.

Then taking the fundamental solution of Helmholtz equation

$$\left\{ F_n(|\vec{r}_n - \vec{r}|) \right\}_{n=1}^{\infty}, \quad (3)$$

where \vec{r}_n are the positions of auxiliary sources. Then, a new system of functions is constructed as follows:

$$\left\{ U_n(|\vec{r}_n - \vec{r}|) \right\}_{n=1}^{\infty}, \quad (4)$$

where: $\left\{ U_n(|\vec{r}_n - \vec{r}_s|) \right\} = W \left\{ F_n(|\vec{r}_n - \vec{r}_s|) \right\}$;

$$|\vec{r}_n - \vec{r}_s| = \sqrt{(x_n - x_s)^2 + (y_n - y_s)^2 + (z_n - z_s)^2}; \quad M(x_n, y_n, z_n) \in \sigma; \quad M(x_s, y_s, z_s) \in S.$$

In [Ref 2,14] it is shown, that the constructed functions satisfy the following conditions:

I. For each function from the system $\{F_n\}$, which satisfies equation (1) one can define a new function WF_n on the surface S , where W is an operator of boundary conditions;

II. The system of functions $\{WF_n\}$ are complete and linearly independents on the surface S on the functional space L_2 ;

III. If the coefficients j_n have been found using the conditions of the best (in the meaning of L_2) decomposition of function WU using the first N functions of the system (4).

$$WU^i(x_s, y_s, z_s) \approx \sum_{n=1}^N j_n U_n(|\vec{r}_n - \vec{r}_s|) \quad (5)$$

then, the approximate solution of the boundary problem (1-2)

$$\tilde{U}^s(x, y, z) = \sum_{n=1}^N j_n F_n(|\vec{r}_n - \vec{r}|) \quad (6)$$

will tend to exact solution $U^s(x, y, z)$ as $N \rightarrow \infty$;

In case of homogeneous, isotropic medium the fundamental solutions of the Helmholtz equation, forming system of function (3), have the following form:

$$F_n(x, y) = H_0^{(1)}(k\sqrt{(x_n - x)^2 + (y_n - y)^2}), \quad 2D \text{ case} \quad (7)$$

$$F_n(x, y, z) = \frac{e^{ikr_n}}{r_n}, \quad 3D \text{ case} \quad (8)$$

$$\text{where } r_n = \sqrt{(x_n - x)^2 + (y_n - y)^2 + (z_n - z)^2}$$

Similar properties are observed for multipole sources considered in [3-5] for solution of boundary problems with cylindrical symmetry:

$$\left\{ H_n^{(1)}(k\rho) e^{in\varphi} \right\}_{n=1}^{\infty} \quad (9)$$

where $H_n^{(1)}(k\rho)$ are first kinds Hankel function of the order n ; the origin of coordinates was at an arbitrary point inside the surface S .

It is necessary to note that in the case of closed surface, when the cross section of the cylindrical body is circular or consists of parts of circular cylinders Hankel functions of higher orders are more convenient. [Ref 3-5,29]. In general case:

$$U(r) = \sum_{n=0}^N \sum_{m=-M}^M a_{nm} H_m^{(1)}(kr_n) \quad (10)$$

Consider the case of 2D scatterer. Then eq (5) turns to:

$$\sum_{n=1}^N j_n W H_0^{(1)}(k\sqrt{(x_n-x)^2+(y_n-y)^2}) = W U^i(x,y), \quad M(x,y) \in L, \quad (11)$$

where L is a contour of cross section of cylindrical scatterer; $M(x_n, y_n) \in l$ are the points on the auxiliary contour l wholly lying inside the area limited by the main contour L .

Properties (I-III) of the fundamental solutions guarantee existence of the coefficients $\{j_n\}_{n=1}^N$, which ensure fulfillment of equality (6) on average. They can be found by the method of orthogonalization [2], method of least squares or by the method of collocation [30].

In the case of the collocation technique, the boundary conditions are satisfied in a finite number of points located on two medium interface, and the number of the collocation points equals or surpass the number of the unknown coefficients. Then

$$\sum_{n=0}^N j_n W H_0^{(1)}(kR_{nm}) = W f(x_m, y_m), \quad (m,n=1,2,\dots,N), \quad (12)$$

where: $R_{nm} = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2}$

Contrary to other techniques the collocation technique allows one to construct a very simple algorithm. Additionally, this reduces the solution of the problem to a system of linear algebraic equations, (which requires minimal computer time), and provides solution to problems with an arbitrary geometry.

In eq. (12) the two sets of points are interchangeable [Ref. 16]. Therefore the points on the inner surface σ can be assumed as the boundary and the auxiliary sources could be moved on the surface S . Then coefficients will be directly proportional to the current distribution along the surface of the body.

The proposed method raises a number of basic questions such as the selection of the form and dimension of the auxiliary contour, the distribution of the collocation points on the basic contour and the auxiliary sources on the auxiliary contour, the number of the auxiliary sources, the optimal choice of the enumerated parameters for the problem stated and the evaluation of the calculation accuracy and its dependence on the auxiliary parameters.

Theoretical investigations and numerical computations have shown that the stability and the speed of convergence of the solution depend on the correct choice of the auxiliary parameters. In the case of an incorrect choice of the parameters the algorithm could diverge.

Each above question peculiarity has been studied and principles answers will be given below. The fundamental results are that the correct choice of the auxiliary contour is very important for the method's efficiency. Consideration of the scattered field singularities, and the 'resonance' of the auxiliary contour are essential. Investigation of these problems allowed us to increase the number of possible applications of MAS.

2. THE REGION OF SCATTERED FIELD SINGULARITIES

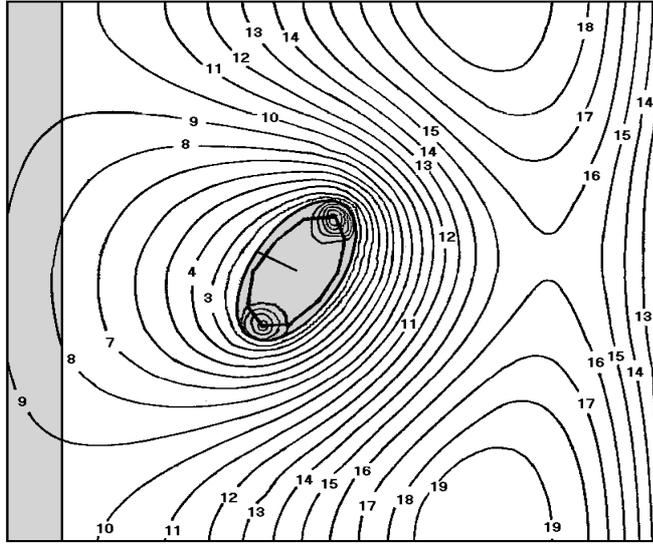


Fig 1. Metallic lattice over a layer of dielectric.

The fundamental idea of this part is based on the fact that every scattered field, which satisfies the radiation condition, must have singularities in some area. A scattered field's function is analytical throughout outside the body's surface. Note, that in the case of a diffusion scatterer, when each part of the surface irradiates independently, random, non-correlatively, it is impossible to continue the function of the scattered field analytically into the nonphysical region. The main singularities are located on the diffusion scatterer surface. The numerical experiment has shown and

it is obvious, that if a current's magnitude and its phase on the surface of the body are smooth, the scattered field can be continued analytically into the nonphysical area in the body, up to the scattered field singularities. They are located inside the scatterer body without the age.

Many researchers have theoretically investigated the details of this problem [6-8,20-27].

To make the method more effective we must locate the auxiliary sources in the singularity region. The matter is that the necessary number of terms of the series (6) strongly depends on the relative distance (along the normal) between the real contour and the contour on which the auxiliary sources are placed. When the auxiliary contour moves away from the real one the number of the terms in (6) decreases strongly and consequently computer time required decreases also. This can be explained by the fact that by shifting the sources into the conducting body the scattered field function becomes more smooth on the surface of the body and its identity with the incident field in the collocation points remains in other points of the contour, i.e. the fulfillment of the boundary conditions in the region between collocation points is improved.

However, the shift of the auxiliary contour is restricted by the location of the scattered field singularities.

The field reflected from an elliptical cylinder edge has phase centers located in the ellipse foci's region. Hence, the scattered field must have divergence near the ellipse foci. In the case of diffraction on the periodical structure, the picture of equal-amplitude lines of the electric field when the lattice elements have elliptical section is shown in Fig. 1. The auxiliary contour passes through the basic contour's foci, and the sources' currents near the ellipse foci reach a large value and, practically, only these currents form the scattered field.

In a limited case, when the ellipse reduces to the strip, the above-mentioned singularity is known as "singularity of the edge". For the case of an elliptical surface case this singularity is expressed weakly and they are located in the ellipse focus region. Thus, for better efficiency of the method, we must locate the auxiliary sources in the scattered field's singularity region, i.e. in convexity centers of the scattered field or in basic contour angles (in the case of a surface with edges).

The scattered field singularity problem has already been studied and will be the main subject of an extend article which is under preparation at present.

3. THE "RESONANCE" OF THE AUXILIARY CONTOUR (solution of interior problems)

During the solution of diffraction problems it has been noted that for some critical values of wavenumber k_a (for example in case of circular cylinder) convergence becomes a worse [12,13]. Investigation has shown that the “critical” values of k exactly coincide with the eigenvalue of the region bounded by the auxiliary contour. At the same time, the fields inside the auxiliary contour describe the eigenfields of the region. Similar patterns of the eigenfields are observed inside the basic contour if the auxiliary contour is out of the region D .

During more detailed investigation [Ref. 30,31] it has been shown that these k_a with high accuracy were in agreement with eigenvalue of inner area. Unlike the other values of k , penetration of electromagnetic field into the inner area is observed, and this field is exactly the eigenfield of the inner contour. That is the reason for sharply changing field values around k_{res} . Results will not be changed in the case of using shifting boundary conditions.

Consider the problem of calculation of eigenvalues and eigenfields of the along - regular waveguide. Usually, such problems are reduced to the solution of a system of homogeneous algebraic equations and further a set of parameters is determined for which this homogeneous system possesses a non-trivial solution. As it is known, the process of investigation of the dispersion properties of various waveguide systems includes the calculation of propagation constants, impedance, losses, etc. This requires the calculation of eigenmodes with large accuracy. Therefore the conventional methods based on a solution of a homogeneous system, in a numerical realization meets difficulties, which become insuperable for high mode of fluctuations.

Firstly - the determinant of a corresponding homogeneous system becomes close to zero in a wide range of parameters and then the search of critical values of the parameters is difficult.

Secondly - the process of computation account of eigenmodes is formidable. The error of calculation grows sharply for high order of modes.

Such problems are met even in the case of a simple along-regular hollow metal waveguide. The problem becomes even more complicated in the case of dielectric waveguide.

On the basis of MAS, a relatively simple method for eigenmode calculations is suggested [Ref. 30,31]. The then problem is reduced to the solution of a non- homogeneous system of linear algebraic equations.

Consider the solution of two-dimensional scattering problem on the basis the Fredholm singular integral equations of the first kind [32] and assume that a monochromatic electromagnetic wave incidents a perfectly conducting cylindrical body with cross section contour L . Then, the determination of diffraction field is reduced to the solution of boundary problem (1-2) in which the operator W - is either the unit operator (E - polarization), or the derivative along the normal to the contour L (H - polarization).

If the solution of the problem is formed as the determination of the currents induced on the contour L as follows:

$$U(x, y) = \int_L J(L) H_0^{(1)}(k\sqrt{(x-x_L)^2 + (y-y_L)^2}) dL, \quad (13)$$

then the problem is reduced to the solution of the following Fredholm singular integral equation of the first kind:

$$\int_L J(L) H_0^{(1)}(k\sqrt{(x^*-x_L)^2 + (y^*-y_L)^2}) dL = E_z^i(x^*, y^*), \quad M(x^*, y^*) \in L. \quad (14)$$

As is known, the given problem is incorrect according Hadamard [Ref. please add]. Furthermore, a solution may not exist for any right hand side of Eq. (14).

Despite Hadamard's restrictions, in those problems of diffraction, which are reduced to the solution of the given integral equation, the mentioned incorrectness does not manifest itself. Firstly in the problems of diffraction, more often, we are interested not in the currents but in a scattered field, in which the rapidly oscillating term $i(L)$ of the current does not change the result:

$$\int_L i(L) H_0^{(1)}(k\sqrt{(x-x_L)^2+(y-y_L)^2}) dL \approx 0. \quad (15)$$

Secondly, the right hand side of Eq. (14) in all practically important problems of diffraction is a smooth function on the contour L , which itself is a solution of Eq. (1) and guarantees the existence of the solutions of the boundary problem (1,2) and of the integral equation (14). This is confirmed by many authors [Ref. please add] where the two-dimensional and three-dimensional scattering problems are solved using Fredholm singular integral equations of the first kind.

From the above considerations a number of important conclusions derives as follows.

The function, determined by formula (13), coincides with an incident field $E_z^i(x, y)$ not only on the boundary of the area D , but also everywhere inside D .

$$\int_L J(L) H_0^{(1)}(k\sqrt{(x_1-x_L)^2+(y_1-y_L)^2}) dL = E_z^i(x_1, y_1), \quad M(x_1, y_1) \in D, \quad (16)$$

where $J(L)$ is the solution of the integral equation (14).

The functional relation of $J(L)$ (16), in the method of "shifted boundary conditions", is considered as a non-singular integral equation [Ref. 16].

We consider some properties of the solution of this equation:

- a) The integral equations (14) and (16) have the same solutions;
- b) If the wave number k coincides the eigenvalue k_i of the Dirichlet's or Neumann's problem for the area D_0 bounded by contour l , then the solution of an equation (16) consists of two terms

$$J_0(L) = J(L) + \omega_1(L), \quad (17)$$

where $J_0(L)$ is the solution of Eq. (16); $J(L)$ is the solution of Eq.(14); $\omega_1(L)$ - is a nontrivial solution of homogeneous integral equation:

$$\int_L \omega_1(L) H_0^{(1)}(k_i\sqrt{(x_1-x_L)^2+(y_1-y_L)^2}) dL = 0. \quad (18)$$

It follows then that the eigenfield of the area D_0 , for the wave numbers k_i is formed as:

$$U_i(x, y) = \int_L J(L) H_0^{(1)}(k_i\sqrt{(x-x_L)^2+(y-y_L)^2}) dL - E_z^i(x, y) \quad (19)$$

Actually the first term of (17) provides compensation of the incident $U_i(x, y)$ everywhere in the area D_0 (since it is the solution of Eq. (14)), and the second term $\omega_1(L)$, as the eigencurrent, describes an eigenfield in this area. Numerical results for the problem of a rectangular waveguide illuminated by plane wave are shown in Fig. 2. The auxiliary contour is a circle of radius $a=1$. The collocation points are located at the square of size $l=a$. The integral equation (16) is numerically solved for wave number $ka = \sqrt{2}$, which is critical for the square. Fig. 2(a) is the superposition of the diffraction field, shown in Fig. 2(b) and the eigenfield of the square, shown in Fig. 2(c).

Using the properties of the integral equation (16) a simple algorithm for calculation of critical frequencies and eigenfields of the along-regular metal and dielectric waveguides is derived.

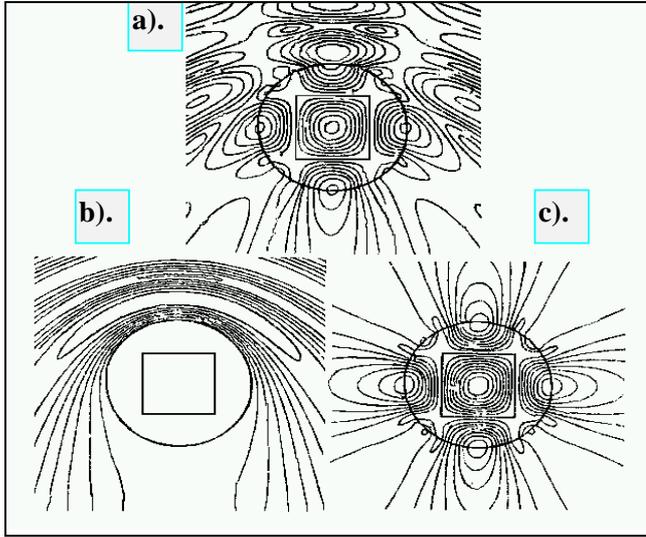


Fig 2. Fields in the near zone of the cylinder.

possibility of investigating them almost simultaneously using same methods and algorithms.

The present method for the solution interior problems must be considered as a mathematical model of the physical experiment in which resonant frequencies and eigenfields of the resonator

Such algorithm was used for the solution of several interior problems. Solution of a great number of applied problems of this kind has shown that this method allows the calculation of eigenmodes with high accuracy. Additionally calculation of high wave oscillation modes does not require extra accuracy or supplementary auxiliary sources. Accuracy of calculation of all modes is the same in contradiction with the 'traditional' methods of solution of homogeneous equations where high order calculations, requires much more calculation efforts.

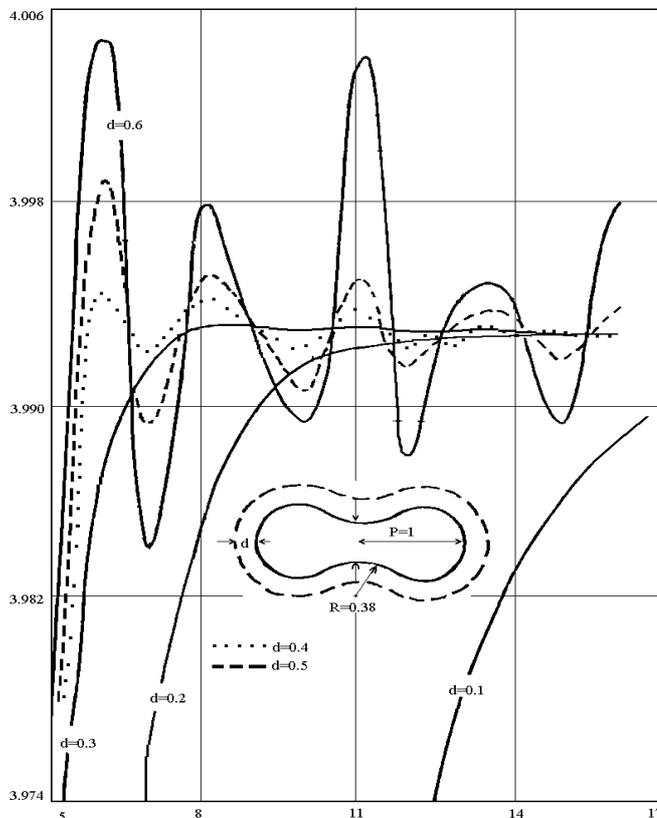
Therefore, according MAS, there is a deep relation between diffraction and eigenvalue problems, and this gives the possibility

of the field's sharp increases in the internal region with the radiation of the system by changing the field's frequency.

Eigenfield's singularities are created and determined only by the main investigated surface and they are located in concavity centers in fracture points of the studied surface (see fig. 3).

The presented method of calculations based on the solution of an inhomogeneous system of linear algebraic equations allows one to determine more precisely the eigenvalues, simplifies the calculations of the eigenfunctions, and does not require any additional increasing of N for computing supreme modes.

Fig 3. Dependence of convergence from the form of auxiliary contour. Minimum radius of curvature of a main contour $R_{\min} = 0.38$. For a distance between the contours $d > R_{\min}$ results diverges



4. ALGORITHM OF THE CONSTRUCTION OF THE SOLUTION OF THE DIFFRACTION PROBLEMS ON THE BASIS OF MAS

In this part the algorithm of the construction of the solution of various problems of diffraction is formulated. We shall formulate some general statements of the algorithm.

As the first step the analysis of the geometry of the scatterer surface is carried out. For simple surfaces, when it is possible to establish the character and location of main singularities, corresponding appropriate auxiliary sources are placed at these points. It provides the most optimal description of the scattered or eigenfield.

If only the area of the location of main singularities is known, the auxiliary surfaces should cover this area including the point of the electrostatic image of the radiator.

Following this important step is the process of the determination of amplitudes and phases of auxiliary sources. The algorithm should provide for a minimum of machine resources taking into account a reasonable mean-square-error.

As was already noted, unknown coefficients are determined by the method of collocation. Such an algorithm reduces a problem to the finding of pseudo-solutions of the following redefined (in a general case) system of linear algebraic equations:

$$\sum_{n=1}^N J_n W F_n (|\vec{r}_n - \vec{r}_m|) = W U^i(x_m, y_m, z_m), \quad (m = 1, 2, \dots, M) \quad (20)$$

where $M \geq N$; $\{x_n, y_n, z_n\}$ - are the points on the auxiliary surface; $\{x_m, y_m, z_m\}$ - are the points of collocation on the surface of the scatterer.

In this algorithm, the value of the mean-square error depends on a number of factors. Mainly, these factors are: the form of the auxiliary surface, the number and the distribution law of the auxiliary sources on it, as well as the number and the distribution law of the collocation points on the main surface.

Computing experiments shows that in this connection it is reasonable to follow the following recommendations:

- a) For smooth surfaces the collocation points should be dispersed uniformly;
- b) The auxiliary surface should pass through the points, which are located at the equal distance from the main surface;
- c) With an increase in distance between the main and auxiliary surfaces the convergence of the results is improved. This distance should not exceed minimum radius of the negative curvature of the main surface;
- d) In the case of edges at the main surface, the account of the special character of the field, in the points of geometrical singularities should result in the computation of the algorithm. It is more advantageous to smooth the edges under certain radius of curvature and to trace out its influence on the final results;
- e) In the study of diffraction on thin opened surfaces and screens the presence of some thickness of the order $kd = 0,01$ should be assumed. Thus the deviation from the true results does not exceed one percent.

Taking account these recommendations the results of calculations monotonely go to the true ones with an increasing number of collocation points. They are general. However, in each

concrete case there should exist set of the most optimal auxiliary parameters providing the given accuracy.

4.1 The investigation of along-regular hollow metal waveguide. As is known, the investigation of dispersion properties and eigenfields at the along-regular waveguides is reduced to the solution of the following homogeneous boundary problem for the Helmholtz equation. Let us find in the area D_0 , bounded by the contour l , a non-trivial solution of the equation

$$\Delta U(x, y) + g^2 U(x, y) = 0, \quad (21)$$

which satisfies the boundary condition

$$WU(x, y) = 0; \quad M(x, y) \in 1, \quad (22)$$

on the contour l , and the spectrum $\{g_i\}$ of those values of the parameter g , for which these solutions exist.

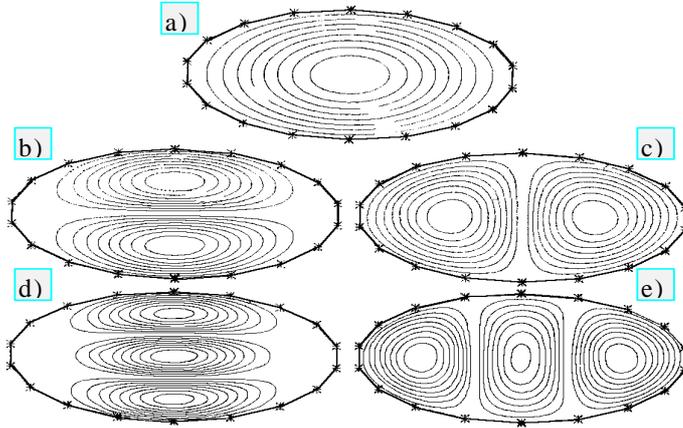


Fig 4. Eigenfields of elliptic waveguide $a/b = 0.5$:

a) Wave of a type E_{01} , $\lambda/b = 1.68$;

b-c) Wave of a type E_{02} , $\lambda/b = 0.91$, $\lambda/b = 1.25$;

d-e) Wave of a type E_{03} , $\lambda/b = 0.63$, $\lambda/b = 0.99$.

The algorithm of calculation of critical frequencies and eigen fields consists in the following:

a) Let us collocation points $\{x_m, y_m\}$, ($m=1,2,\dots, N$) on the main contour l ;

b) Let us construct such an auxiliary contour L , which completely surrounds the main area D_0 , and distribute on it N centers of approximation $\{x_n, y_n\}$, ($n=1,2,\dots,N$), of the integral operator in (16) according to 3.b;

c) Define the auxiliary function $U_i(x, y)$, as some known, particular solution of the boundary problems (21-22) (for instance, the function describing the field of the linear source, located outside of the area D_0);

d) Rewrite the integral equation (16) in the equivalent form of the system of linear algebraic equations:

$$\sum_{n=1}^N J_n W H_0^{(1)}(g \sqrt{(x_n - x_m)^2 + (y_n - y_m)^2}) = W U^i(x_m, y_m) \quad (23)$$

e) Solve the system (23) for various values of the parameter g and trace out the behavior of the function:

$$U(x_0, y_0) = \sum_{n=1}^N J_n \text{WH}_0^{(1)}(g \sqrt{(x_n - x_0)^2 + (y_n - y_0)^2}) - U^i(x_0, y_0) \quad (24)$$

in some point $M(x_0, y_0)$ of the area D_0 . Find those values of the parameter g for which this function reaches its relative maximum.

Just these values of the parameter g will correspond to the eigenvalues of the given area, and the function (24) for these g will describe the eigenfield (see. Fig. 4). For a given case five auxiliary sources and five points of a collocation provide accuracy of calculation on the order of 0.1 % (taking into account symmetry).

4.2 The investigation of along-regular dielectric waveguide. Let us investigate dispersion properties of an along-regular dielectric waveguide with a contour of cross section \mathbf{l} and dielectric permittivity ε_1 , placed in a homogeneous, isotropic medium with permittivity ε_2 .

As was shown in [31], the problem can be formulated in non-traditional transverse components, choosing as a potential function one of the following pairs: either (H_x, H_y) or (E_x, E_y) . In the same works the advantages of such an approach are discussed.

This boundary problem has the following form: Find a non-trivial solution of the wave equations:

$$\begin{aligned} \Delta H_{1x,y} + g_1^2 H_{1x,y} &= 0, \quad M(x, y) \in D_1; \\ \Delta H_{2x,y} + g_2^2 H_{2x,y} &= 0, \quad M(x, y) \in D_2; \end{aligned} \quad (25,26)$$

satisfying the boundary conditions on the contour \mathbf{l} :

$$\begin{aligned} H_{1x} &= H_{2x}; \\ H_{1y} &= H_{2y}; \\ \frac{1}{\varepsilon_1} \left[\frac{\partial H_{1x}}{\partial y} - \frac{\partial H_{1y}}{\partial x} \right] &= \frac{1}{\varepsilon_2} \left[\frac{\partial H_{2x}}{\partial y} - \frac{\partial H_{2y}}{\partial x} \right]; \\ \frac{\partial H_{1x}}{\partial y} - \frac{\partial H_{1y}}{\partial x} &= \frac{\partial H_{2x}}{\partial y} - \frac{\partial H_{2y}}{\partial x} \end{aligned} \quad (27)$$

Let us also find a set of parameters k and h , for which these solutions exist. Here $g_1^2 = \varepsilon_1 k^2 - h^2$ and $g_2^2 = \varepsilon_2 k^2 - h^2$, and conditions $\text{Re } g_2 = 0$; $\text{Im } g_2 > 0$ provide the absence of the energy radiation in the transverse direction.

In a given problem two auxiliary contours L_1 and L_2 are constructed. The contour L_1 completely lies in the area D surrounded by the contour \mathbf{l} and the contour L_2 , which completely surrounds this area.

We look for the solution in the following form:

$$\begin{aligned}
U_1(x,y) &= \oint_{L_1} a(L) H_0^{(1)}(g_1 r_1) dL_1 - f_1(x,y); \\
U_2(x,y) &= \oint_{L_1} b(L) H_0^{(1)}(g_1 r_1) dL_1 - f_2(x,y); \\
U_3(x,y) &= \oint_{L_2} c(L_2) K_0(-ig_2) dL_2; \\
U_4(x,y) &= \oint_{L_2} d(L_2) K_0(-ig_2) dL_2.
\end{aligned} \tag{28}$$

Where $a(L_1)$, $b(L_1)$, $c(L_2)$, $d(L_2)$ are unknown density of potentials distributed on the contours L_1 and L_2 correspondingly; $f_{1,2}(x,y)$ are the known particular solution of a equation (26).

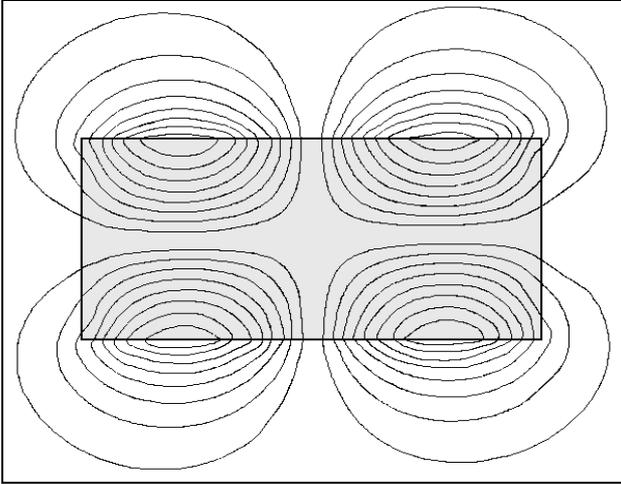


Fig. 5. An eigenfield of an Electric type of rectangular dielectric waveguide. (longitudinal component of H); $2a=3.35$ mm, $2b=6.7$ mm, $\epsilon=2.08$, $kb=4.2097$; a slowing down coefficient of $\zeta=1.177$.

The algorithm of the solution of the received system of integral equations of the first kind is similar to that stated in the previous item. The main contour L is divided uniformly in N points of collocation. The integral operator is replaced by a sum and a non-homogeneous system of linear algebraic equations for various values of the parameters k and h is solved. The values of the function (28) in some points inside the area D are nearly equal to zero if $h \neq h_{\text{eig}}$ and $k \neq k_{\text{eig}}$ (simultaneously). But for critical values of these wave numbers (28) describe eigenfields of a given dielectric waveguide (see. fig 5). As in the previous problem, five or six auxiliary sources and points of collocation provide an accuracy

of calculation of the order of 0.1 % (taking into account the symmetry).

4.3 Diffraction of electromagnetic waves on conducting bodies. These problems are conventional for MAS. This method was first approved just in such problems. There are plenty of works in which two-dimensional problems of diffraction on isolated or periodic structures or rotation bodies, as well as rather complicated three-dimensional problems, are solved. Mainly, when the algorithm described is used, it is possible to reach a high accuracy of calculation and to perform the investigations for a wide range of variation of the wave number, from electrostatics up to ka of the order - 200.[34]

Let us consider some examples. In the two-dimensional problem the boundary conditions have the form $\mathbf{E}_\tau = \eta \mathbf{H}_\tau$, (in general case, where $\eta = (1-i)\delta / \sigma$ - is surface impedance):

$$\alpha \frac{\partial U(x,y)}{\partial n} - ik\beta U(x,y) = 0, \quad M(x,y) \in L, \tag{29}$$

where L is the contour of the cross section of cylinder; δ - is the thickness of the skin-layer; σ - is the conductivity;

$$\left. \begin{aligned}
 \alpha = 0, \quad \beta = 1, \text{ for perfectly conducting body;} \\
 \alpha = \frac{\eta}{120\pi}, \beta = 1, \text{ for the impedance body;} \\
 \beta = 0, \quad \alpha = 1, \text{ for perfectly conducting body;} \\
 \beta = \frac{\eta}{120\pi}, \alpha = 1, \text{ for the impedance body;}
 \end{aligned} \right\} \begin{array}{l} \mathbf{E} \text{ - polarization;} \\ \mathbf{H} \text{ - polarization.} \end{array}$$

For isolated bodies function $U(x,y)$ has the form (7). For periodic structures the functions describing the fields of auxiliary sources should have the form:

$$F_n(\vec{r}_n - \vec{r}) = \sum_{m=-\infty}^{\infty} \alpha H_0^{(1)}(k\sqrt{(x-x_n)^2 + (y-y_n-md)^2}). \quad (30)$$

Further the singularities are singled out in an explicit form and use the Poisson formula [35] is used for the improvement of convergence of this series. Here d - is the period of the structure; $\alpha = 1$ for diffraction lattices, and $\alpha = (-1)^m$ for waveguides with non-homogeneous filling.

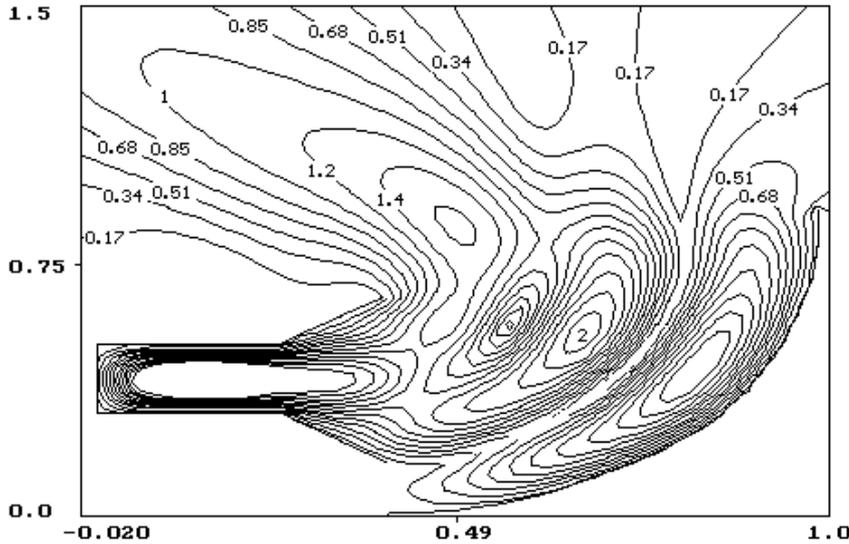


Fig. 6. Excitation of the conducting screen by horn.

In fig. 6 results of calculation for the following problem are indicated: a linear current excites a waveguide with a horn and the radiated wave is scattered by some disconnected screen. Varying the location of a probe, we achieve the best matching of the system with external space, and varying the geometry and impedance of the screen and flange of the horn, we form the best diagram of radiation. At the same time interaction among all the ele-

ments of the system is taken into account.

4.4 waveguide Filter. Consider a microwave filter in the form of rectangular waveguide with transverse plates at the electrical wall. The given problem is reduced to the diffraction of the waveguide wave H_{01} on the multilayer lattice with elements in the form of plates of rectangular cross-section. Here d is the size of the wide wall of the waveguide.

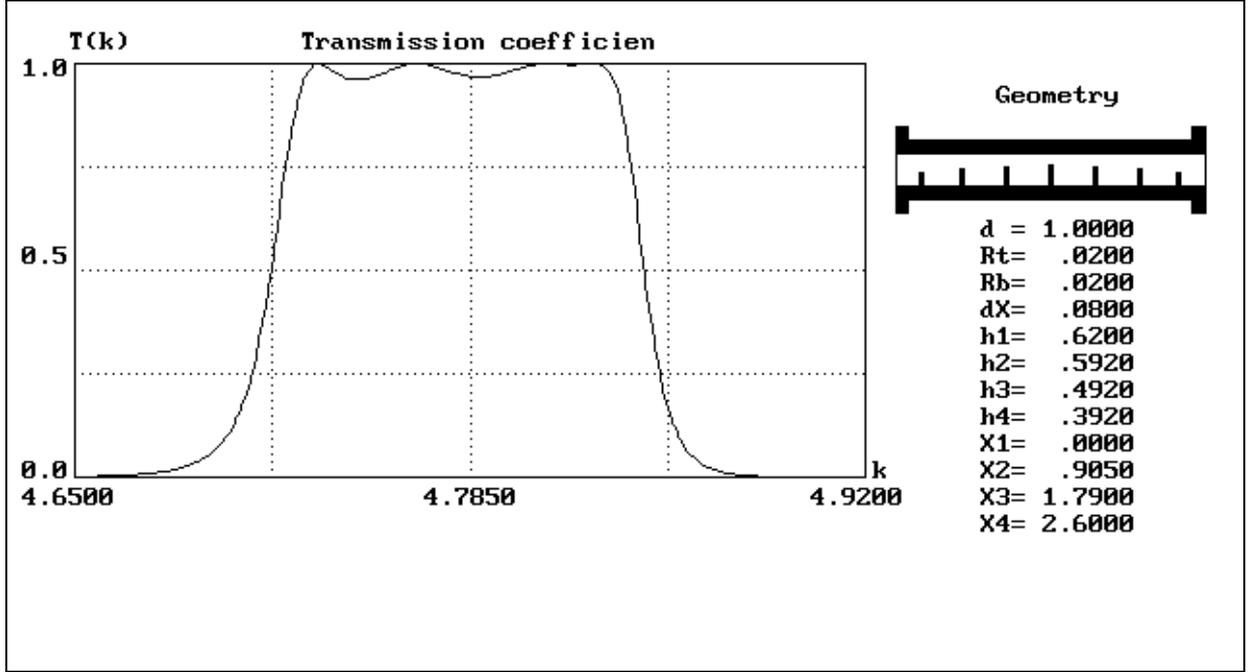


Fig 7. The waveguide filter

For the formation of the geometry of a modern microwave filter with the given characteristics, one needs to have the possibility to vary such parameters as:

- a) number of plates (from 1 up to 7);
- b) thickness of plates (from $dx = 0.05d$ up to $dx = 0.5d$);
- c) heights of plates (from $h=0$ up to $h=d$);
- d) rounding radius of the rectangular edge of plate ($Rt = 0.02d$);
- e) rounding radius in the points of contacts of plates with the short walls of the waveguide ($Rb = 0.01d$).

The last two parameters are technological.

In fig.7 the dependence of the transmission coefficient $T(k)$ of the microwave filter with given parameters on the wavenumber of the incoming wave H_{01} is indicated.

In a problem of excitation of rotation bodies, determining the choice of the form of auxiliary sources and their location is important. Depending on the geometry, various auxiliary sources can appear to be optimal. The optimal number of sources ensuring the given accuracy of calculation strongly depends on their form.

We shall give some examples.

4.5 For a convex rotation body extended everywhere, the auxiliary sources of the form (8) (dipole) located on the rotation axis inside the surface most optimally describe the scattered field. For extended rotation bodies with sign-varying curvature, it is reasonable to use the same sources (8) with the singularities in the complex plane:

$$F_n(\vec{r}_n - \vec{r}) = \frac{e^{ikr_n}}{r_n}, \quad r_n = \sqrt{x^2 + y^2 + [z - (z'_n \pm z''_n)]^2} \quad (31)$$

Is easy to check that the singularities of the function (31) are “smeared” on a circle of radius z''_n . This property allows one to describe the main singularities of the field scattered by such bodies. In this case the singularities can appear not only on the rotation axis.

In problems of excitation of oblate rotation bodies, auxiliary sources in the form of spherical multiples [19] can be successfully used. Consider the bodies formed by the rotation of some

flat contour L around the Z - axis, which lies in the same plane but does not pass through the given contour (torus with meridian section in the form of contour L).

Assume that the contour L is in the ZOx -plane and the rotation occurs around the Z-axis. For such rotation bodies, the auxiliary sources can have the form:

$$\vec{F}_n(\vec{r}_n - \vec{r}) = a_n \int_0^{2\pi} \vec{e}(\alpha) \frac{e^{ikr_n}}{r_n} d\alpha, \quad (32)$$

$$r_n = \sqrt{(x - a_n \cos \alpha)^2 + (y - a_n \sin \alpha)^2 + (z - z_n)^2}$$

$M(a_n, \alpha, z_n)$ - is the point on the torus, which is formed by the rotation of the auxiliary contour l (see item 3a-3b); $\vec{e}(\alpha)$ - is the unit vector parallel to a current in the corresponding point of the main surface.

The potential of the scattered field has the form:

$$\vec{U}(\rho, z) = \sum_{n=1}^{N+1} J_n \vec{F}_n(\vec{r}_n - \vec{r}), \quad (n = 1, 2, \dots, N + 1), \quad (33)$$

where N is the number of points on the auxiliary contour l; and $M(x_{N+1}, y_{N+1})$ - is the point of the electrostatic image of the irradiator on the surface of the scatterer.

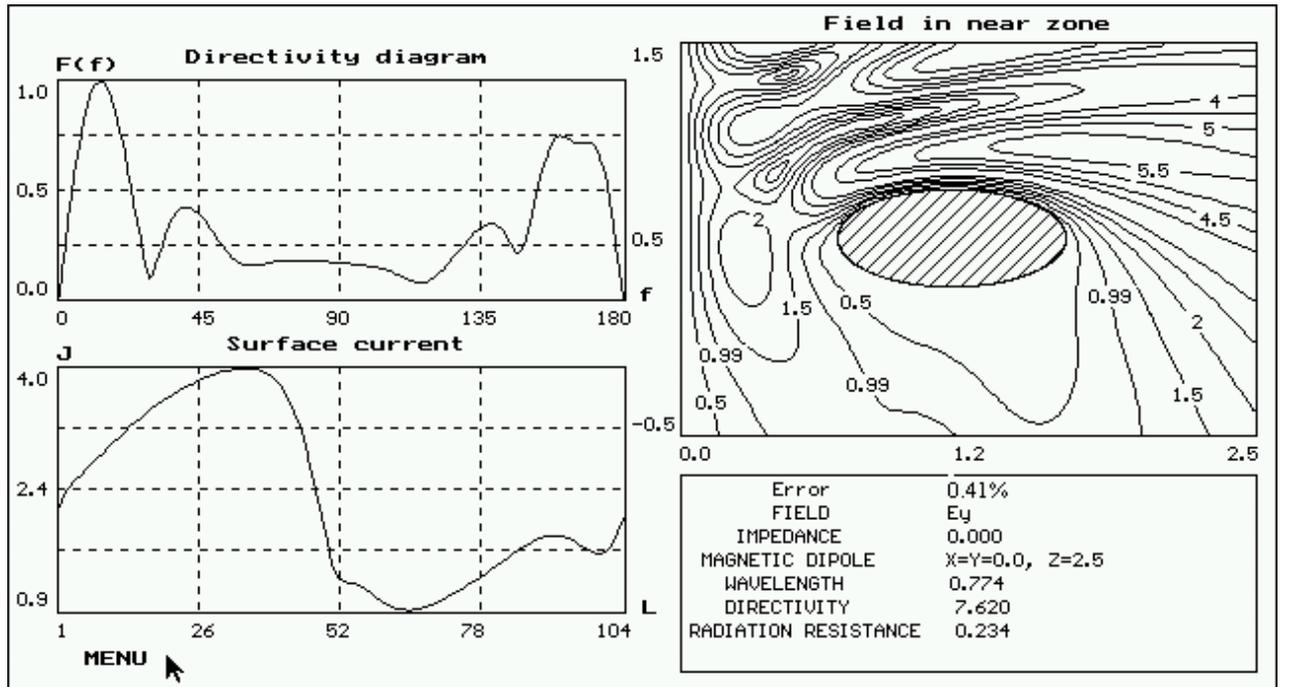


Fig 8. Excitation of the elliptic torus by magnetic dipole

In the fig. 8. results of calculation of the excitation of torus with elliptical meridian section by the magnetic dipole located in the point $x=0, y=0, z=0.2k$ and parallel to the Z-axis are given. The centre of the ellipse is shifted from the rotation axis on distance $kd=1.2$ and the semi-axis of ellipse are correspondingly $ka=10.0$ and $kb=5.0$. The auxiliary contour l surrounds the foci of the given ellipse.

4.6 Diffraction of electromagnetic waves on anisotropic, absorbing dielectric bodies.

In the investigations of the problem of excitation of dielectric bodies it is necessary to have two

various systems of auxiliary sources. The first of them should be constructed by the fundamental solutions of the Helmholtz equation for the external area and should satisfy the radiation condition. The second should be constructed by the fundamental solutions of the wave equation for the internal area. The auxiliary sources which describe a scattered field, can have form (7-9), or (30) for periodic structures. The corresponding auxiliary surface should surround all the main singularities of the scattered field.

The auxiliary sources which describe an internal field, can have the form:

$$F_n^l(|\vec{r}_n - \vec{r}|) = J_0(k|\vec{r}_n - \vec{r}|), \quad \text{or} \quad F_n^l(|\vec{r}_n - \vec{r}|) = \frac{\sin(k|\vec{r}_n - \vec{r}|)}{k|\vec{r}_n - \vec{r}|} \quad (34)$$

The appropriate auxiliary surface can be inside as well as outside of the body, because the functions (34) have no singularities. If the dielectric is homogeneous or anisotropic with absorption, then we should use a system of functions from the fundamental solutions of wave equations for the corresponding medium. In [33], the corresponding functions (for the two-dimensional and three-dimensional cases) for media with diagonal tensor of electrical permittivity and magnetic permeability are given. For the anisotropic cylinder, which is parallel to the Z-axis, we can write:

$$F_n^c(|\vec{r}_n - \vec{r}|) = J_0(k\sqrt{\varepsilon_z}\sqrt{\mu_x(x-x_n)^2 + \mu_y(y-y_n)^2}), \quad (35)$$

$$F_n^m(|\vec{r}_n - \vec{r}|) = J_0(k\sqrt{\mu_z}\sqrt{\varepsilon_x(x-x_n)^2 + \varepsilon_y(y-y_n)^2}); \quad (36)$$

for \mathbf{E} and \mathbf{H} of polarizations accordingly, where ε and μ , are complex in the general case.

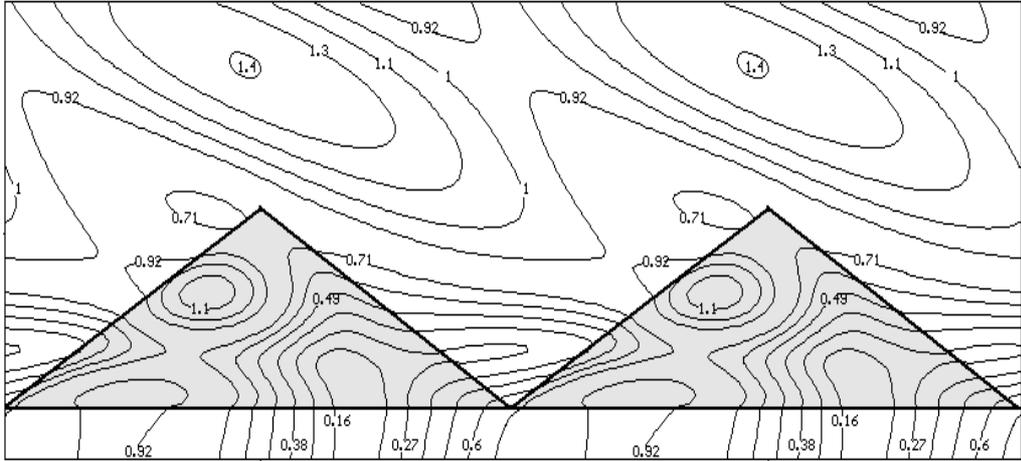


Fig 9. Diffraction of the plane wave on the periodic dielectric layer from triangular elements. $\alpha = 213^\circ$ - angle of incidence; H - polarization; $d=1.0$ - the root of triangle; $h=0.8$ - the height; $\varepsilon'_x = 1.5$, $\varepsilon''_x = 0.2$; $\varepsilon'_y = 2.0$, $\varepsilon''_y = 0.3$; $\mu'_z = 3.0$, $\mu''_z = 0.0$.

In fig. 9 the results of calculations for the periodic cylindrical lattice, which consists of anisotropic absorbing elements, are indicated.

CONCLUSION. The given recommendations for the solution of a diffraction problem on the basis of MAS are most general. However, in each concrete problem, there is a most optimum algorithm ensuring a given accuracy with a minimum cost of machine resources.

The results of calculation are obtained by the help of a computer complex the programs, the basis of which is MAS. The algorithm is constructed according to the above-mentioned scheme. This general algorithm allows one to perform computer experiments for the study of: a) influence of initial and boundary conditions on character of diffraction and propagation of electromagnetic waves; b) influence of the character of excitation, geometry and material of a scatterer on the diffracted field; c) contribution of separate parts of the surface of the body to the scattering diagram and to the near field; d) processes of formation of resonance fields in various waveguide structures; e) processes of propagation of waves in various anisotropic and absorbing media.

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