## A METHOD OF AUXILIARY SOURCES IN APPLIED ELECTRODYNAMICS

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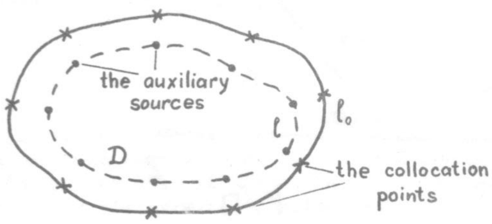
**ABSTRACT**

A method of auxiliary sources based on the distribution of singularities of the basic functions inside the scattering objects where the peculiarities of the analytical continuation of a scattered field are concentrated has been developed. Consequently, by collocation technique the coefficient of multiplying has been determined. A general approach to solving the internal and external problems of applied electrodynamics has been suggested and substantiated.

A method of solution of the boundary problems of mathematical physics has been suggested in [l]. Let's explain the essence of this method in two-dimensional case. If we choose in the domain $D$ some closed contour $l$ and on this $N$ auxiliary sources (see Fig. 1), described by the fundamental solutions of the Helmholtz equation with unknown complex coefficients, the scattered field out of $D$ can be sought in the form of sum of the fields of these sources

$$U = \sum_{n=0}^{N} a_n H_0(kR_n) \tag{1}$$

where $R_n = \sqrt{(x - x_n)^2 + (y - y_n)^2}$; $x$, $y$ are the coordinates of the point of observation; $x_n$, $y_n$ are the coordinates of the auxiliary sources; $H_0$ is cylindrical Hankel function. Each member of (1) satisfies the Helmholtz wave equation and the radiation condition.




Physically it means a replacing the field of the continuous surface by the field of the auxiliary sources on 1. Approaching of the contour 1 to the main one ($l_0$) and increasing of number of sources $N$ results in the well-known field integral presentation

$$U = \int a(s) H_0(kR_s) ds \tag{2}$$

However, removal but not approaching of the contour $l$ from the contour $l_0$ was essential condition for the improvement of the convergence of the calculating process.

In the case of consideration of the dielectric bodies, the fields should be determined inside the $D$ region as well. For this, it is convenient, in each homogeneous part of space as sought function, to choose for example electrical Hertz vector and present it as follows

$$\Pi_1 = \sum_{n=0}^{N} a_n H_0(kR_n) \qquad R_n \notin D$$
$$\Pi_2 = \sum_{n=0}^{N} b_n H_0(kR_n) \qquad R_n \in D \tag{3}$$

The other components of the electrical and magnetic fields can be found by the known correlations. The unknown complex $a_n$ and $b_n$ coefficients can be determined from the boundary conditions.

The choosing of technique of the unknown coefficients determination is the main problem after the selection of the basis functions.

The comparison of the techniques of the unknown coefficients determination has been carried out. Three techniques (1. method of orthogonalization, 2. method of least squares, 3. method of collocation) have been considered for this and a collocation technique turned out to be most effective. It allows to construct a very simple algorithm [2] and enables: 1) to reduce the problem solution to the solving of system of the linear algebraic equations, 2) to solve the problems for noncoordinate bodies.

In the case of using collocation technique the boundary conditions are satisfied in the finite number of the points and number of the collocation points equal the number of the unknown coefficients

$$\sum_{n=0}^{N} a_n L H_0(kR_{mn}) = Lf(x_m, y_m)$$
$$n = 0,1,2,\ldots N \qquad m = 0,1,2,\ldots N \tag{4}$$

$R_{mn} = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2}$; $L$ is operator of the boundary conditions.

The method under consideration has some of peculiarities, introduced as the auxiliary parameters. They are: 1) the form and dimensions of the $l$ contour; 2) the procedure of distribution of the collocation points on the $l_0$ contour and the auxiliary sources on $l$; 3) the number of auxiliary sources. At the same time the following should be considered:
1) an optimum of choosing of the enumerated parameters for the state problem,
2) an evaluation of the calculation accuracy and its dependence on the auxiliary parameters.

The realization of the method has showed that the stability, the rate of condition and the speed of convergence of solution greatly depend on the correct choosing of the auxiliary parameters. In the case of incorrect choosing of the parameters a computing process can be diverged.

Thus, the correct choosing of auxiliary contour is very important for the method efficiency. At that, is essential: 1) the consideration of the singularities of the scattered field, 2) the "resonance" of the auxiliary contour.

The examination of the field values convergence while the distance between the centers of the singularities and the main contour is varied, showed that the removal of the $l$ contour from the $l_0$ decreases in number of the members in expression (1) and the satisfaction of the boundary conditions improves.

The removal of the $l$ contour is limited by the location of the scattered field singularities. Thus, the dependence of solution convergence in the case of elliptical cylinder on the correlation of the cylinder semi axes (coefficient of compression) and the sizes of the auxiliary contour has been shown.

For each value of compression coefficient exists such removal $l$ contour from $l_0$, when the solution results are diverged. An analysis of the numerical results shows that for the best convergence the auxiliary sources must be located in the region of the singularities.

A location of the singularities can be depended also on the character of incident field. Let's consider for example the scattering problem of the two cylindrical sources field if they are located on opposite sides from the round cylinder ($ka$=4.0) at the distances 1.5$a$ (see Fig. 2*a*) and 2.0$a$ (see Fig. 2*b*). Here $a$ is radius of a cylinder and $k = 2\pi / \lambda$ is a wave number. The pattern of the equal-amplitude lines shows the image of two real sources inside the cylinder (non-physical region), i.e., the location of the secondary scattered field centers is determined by geometry and doesn't depend on the incident wavelength.

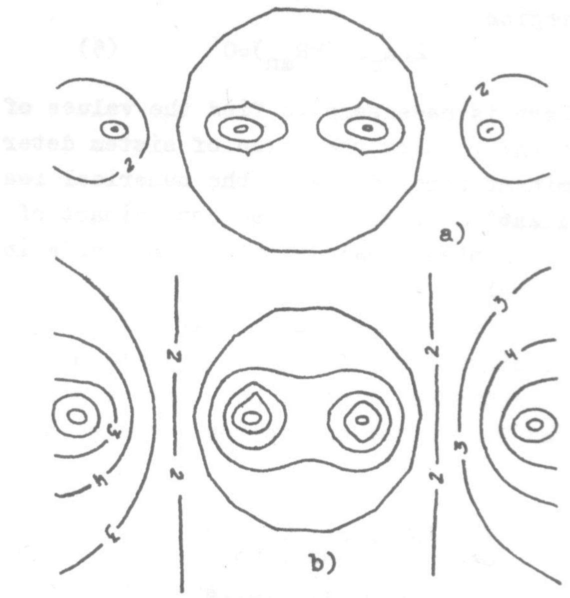



Fig.2a,b

Consequently, the considered results are true within all the range of the frequencies.

The study of the auxiliary contour "resonances" gave the possibility to solve the eigen value problems. Consideration of diffraction problem on a round cylinder shows that for some of "critical" values of the relative perimeter $ka$ ($a$ is the radius) the value of the scattered field greatly differs from the theoretical value. The studies have shown that the "critical" values of $ka$ exactly agree with the eigen value of the region bounded by the auxiliary contour. At the same time the fields inside of the $l$ contour describe the eigen fields of this region. Analogical patterns of the eigen fields are observed inside the $l_0$ contour if the $l$ is out of region $D$.

These results enable to construct a simple algorithm for computing of the eigen fields and the propagation constants of the waveguide systems.

The problem of calculation of the eigen numbers and eigen functions is reduced to the determination of the steady-state oscillation of the region. Usually, the solving of the homogeneous system of the linear equations is used for determination of the eigen values of region

$$\sum a_n H_0(kR_{mn}) = 0 \qquad (5)$$

Here is necessary to find the values of $k$ which doing the value of system determinant zero. However, the numerical realization shows that the determinant of the system equals zero within a wide interval of $k$ alteration.

It is common knowledge that the consideration of eigen oscillation requires the solution of homogeneous system and the forced oscillation requires the solution of unhomogeneous system. If a determinant of homogeneous system equals zero we observed the eigen oscillations. For the unhomogeneous system we have a "resonance" at analogical situation.

Therefore, there is a deep relation between the diffraction and the eigen value problems and this gives the possibility to investigate them from the point of united position by the united method.

The method of calculation of the waveguide systems based on the solution of unhomogeneous system of the linear algebraic equations: 1) allows to more accurate determine of the eigen values; 2) simplifies the calculations of the eigen functions; 3) doesn't require any additional increasing of calculation accuracy for computing supreme type of the waves.

Thus, the primary suggestion method for solving of the diffraction problems on the base of the pure theoretical suppositions has been succeeded in: 1) concretising it, 2) specifying the general states, 3) widening the limits of the application and 4) developing the general approach for the applied electrodynamic problems solving.

In conclusion, let's enumerate some problems, which has been solved by the method of auxiliary sources:

- two-dimensional open and closed resonators with nonhomogeneous dielectrical filling and semi-transparent envelope;
- diffractional properties of a dielectrical body in the presence of a media dividing boundary;
- multilayer grids with complicated cross-sections of grid elements in the presence of a flat-parallel layer of dielectric;
- hollow and coaxial dielectrical waveguides of a complicated profile.

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