

THE METHOD OF AUXILIARY SOURCES IN ELECTROMAGNETIC SCATTERING PROBLEMS

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1. INTRODUCTION

A conventional Method of Auxiliary Sources (MAS) is a method of finding the solution of the boundary problem for a given differential equation by expanding it in terms of fundamental [1-3], or other singular [4-6] solutions of this equation. The special choice of the set of expansion functions is a characteristic feature of the MAS that distinguishes it from conventional variational methods, in which every expansion function *a priori* satisfies the boundary conditions, but does not satisfy the initial differential equation.

The basis of MAS has been formed as a result of a long elaboration of mathematical ideas. The Georgian mathematicians V.Kupradze, M.Aleksidze and I.Vekua were the first who introduced and proved the usefulness of interchanging the differential equation and boundary conditions [7-9] and applied this concept to the solution of specific boundary problems. Here are only some of them: 3-D problems of acoustic diffraction [10], hybrid problems for the equations of elliptic [11] and parabolic type [12,13], elasticity boundary-contact problems in inhomogeneous media [14], biharmonic problems [15], general type of diffraction problems for hydrodynamic and Maxwell's equations in inhomogeneous media [16], certain boundary electrodynamic problems [17,18], etc. "The common rationale of these works is a basic theorem of the completeness in $L_2(S)$ of the totality of denumerable infinite set of the particular solutions generated by the chosen fundamental" [7] or other singular solutions.

The name of MAS currently used did not appear at once. The authors themselves adhered to the names: "The Method of Generalized Fourier Series" (MGFR) [1-3,7], "The Method of Expansion in Terms of Metaharmonic Functions" (METMF) [4-6] and "The Method of Expansion by Fundamental Solutions (MEFS) [8,9]. In earlier works, the solutions of the boundary problems were represented by the generalized Fourier series which coefficients could be calculated in explicit form. However, this required an orthonormalization of the sets of fundamental solutions beforehand. From the mathematical point of view, these solutions were flawless. However, the performed numerical calculations [8,19-21] showed at once non-optimality of preliminary orthonormalization of the set of expansion functions. On the other hand, expanding the solution in terms of the sets of non-orthogonal functions in conjunction with collocation method for determining expansion coefficients appeared to be close to optimal [22]. The sets of fundamental solutions satisfy the conditions above exactly [8]. Thus, the use of this set for construction of the solution combines the efficiency of the approach with optimality of its implementation.

In spite of that mathematical basis of the MGFR, the METMF and the MEFS was formed as early as in 1953-1967 [6,24,25] and in 1967 [23], the first numerical calculations performed in the field of applied electrodynamics appeared somewhat later. However, by the mid 80s the main electrodynamic boundary problems had been tested. Those were electromagnetic scattering and diffraction problems upon perfect conducting and dielectric cylinders and the bodies of rotation embedded in free space or near the interface between the

different media [19,21,26-39], periodic gratings [20,40], longitudinally-regular hollow, dielectric and inhomogeneous waveguides [41-43], etc.

However, the above studies revealed that the problems with algorithm convergence arise for arbitrarily chosen auxiliary surface in the MEFS (MAS) [19-21,45,45] or the location of unified radiation center in the METMF [27,45-48] (the so called Rayleigh hypothesis). Later, it turned out, that the reasonable approximation to the exact solution for each fixed geometry and incident wave could be reached only owing to the proper choice of the auxiliary surface or arrangement of the radiation centers (the latter was pointed out in [6] and implemented in [27,32,38,49]). The performed studies initiated the more detailed elaboration of one of the fundamental properties of scattered fields, namely of their main singularities [45,45]. As a result, some general recommendations have been offered to construct the MAS solution of rather intricate applied problems, which allow their simple numerical implementation [43-45]. Application of the MAS to the objects of complicated shape and complex filling has been also considered [49-63] (the authors do not pretend the list of cited references is complete).

This work gives a conventional interpretation of MAS applied to electromagnetic scattering problems. It offers general recommendations for its implementation and illustrates its application to particular problems for a single and a set of bodies made of various materials, through numerical simulations in a wide frequency band starting from the quasi-static, up to the quasi-optics.

2. PROBLEM FORMULATION

Electromagnetic scattering problems are some of the most important problems in applied electrodynamics. These problems arise in the study of various physical processes in antenna and waveguide theory, radiolocation, meteorology, microelectronics, defectoscopy and other science and engineering branches. From the physical point of view, these problems appear when investigating propagation of waves in media with any discontinuities (Fig. 1) and should then be formulated as appropriate boundary problems of electrodynamics. The main aim in the analysis of these problems is to find the vectors of the secondary electromagnetic field $\{\vec{E}_i, \vec{H}_i, \vec{D}_i, \vec{B}_i\}$ in each domain D_i with different material properties, while the primary field of electromagnetic sources $\{\vec{E}^0, \vec{H}^0, \vec{D}^0, \vec{B}^0\}$ is given.

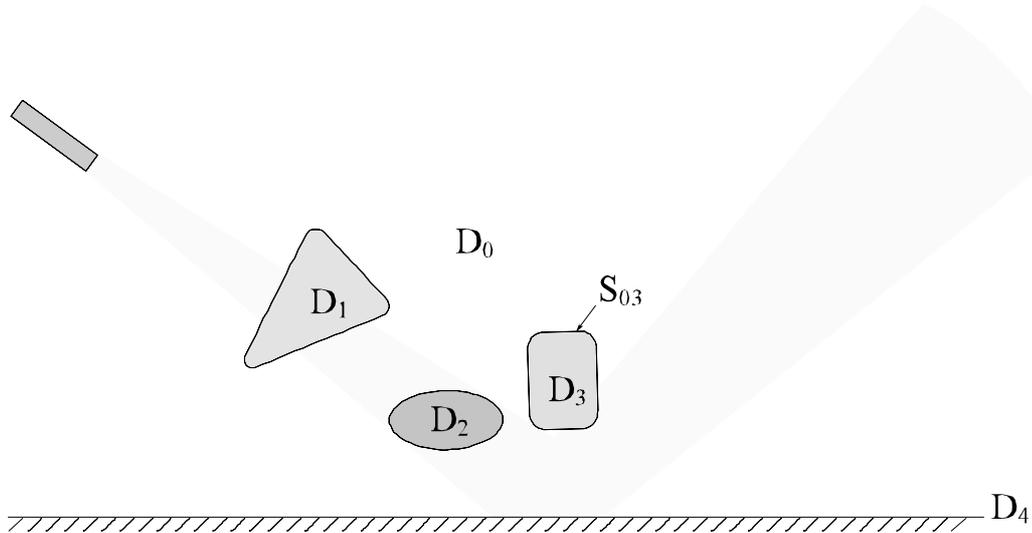


Fig. 1. General geometry of the problem

To formulate the boundary problem, the constitutive equations in each domain D_i , as well as the values of material parameters used should be specified. To a certain extent, most of the well-known media belong to the special cases of the general four-parameter bianisotropic medium, described by the following constitutive relations [64,65]

$$\vec{D} = \varepsilon \vec{E} + i\alpha \vec{B}, \quad \vec{H} = i\beta \vec{E} + \mu^{-1} \vec{B}. \quad (1)$$

In general case, the material properties of the medium are determined by tensors of permittivity ε , permeability μ and magnetoelectric admittances α and β (hereinafter, an $\exp(-i\omega t)$ time dependence is assumed). When $\alpha = \beta = 0$, then this case corresponds to the general type of anisotropic medium, while for the scalar parameters ε , μ , α и β the general case of biisotropic medium is described. The latter case involves subcases of isotropic magnetodielectrics when $\alpha = \beta = 0$, chiral medium when $\alpha = \beta \neq 0$ and Tellegen medium when $\alpha = -\beta \neq 0$.

In the general case, formulation of a scattering problem for a set of bodies with known material properties involves writing electrodynamic equations and constitutive relations in each of the domains D_i with different properties, as well as boundary conditions on the interfaces S_{ij} between the neighbouring domains. However, the general scattering problem can be formally replaced by a few more simple boundary problems for separate domains D_i , solution of which allows of restoring scattered field in a whole space. Therefore, without restriction of generality, consider formulation and solution of the boundary problem for any arbitrary domain D specifying, along with electrodynamic equations, only the constitutive relations, wave equation for the potential function of unknown field and the behavior of this function on the boundary (boundary conditions).

The boundary problem considered for the domain D surrounded by the surface S is reduced to the solution of a wave equation

$$\hat{L} \tilde{U}(\vec{r}) = 0 \quad (2)$$

in the domain D with constitutive relations of (1), where \hat{L} is a wave operator, and $\tilde{U}(\vec{r})$, $\vec{r} \in D$ is an unknown potential function determining uniquely vectors of scattered field and satisfying on S the following boundary conditions

$$\hat{W} \tilde{U}(\vec{r})|_{\vec{r}=\vec{r}^S} = \tilde{f}(\vec{r}^S), \quad M(\vec{r}^S) \in S. \quad (3)$$

Here \hat{W} is an operator of boundary conditions, and $\tilde{f}(\vec{r}^S)$ is the given function determining the behavior of an unknown scattered field on the boundary. For the exterior domains extending to infinity, the unknown field should also satisfy the radiation condition.

The solution to the boundary problem (2)-(3) in the domains with different constitutive relations is the aim of the forthcoming analysis.

3. CONSTRUCTION OF THE SOLUTION BY THE METHOD OF AUXILIARY SOURCES

3.1. Basic statements

In this section, a construction of the solution to the boundary problem (2)-(3) by the conventional Method of Auxiliary Sources (MAS), or MEFS [8,9] is considered.

To solve the boundary problem (2)-(3) in the domain D bounded by the surface S , let us enclose this domain by auxiliary surface S' and distribute on it uniformly the set of points

$\{\vec{r}_n\}_{n=1}^{\infty}$ (Fig. 2). Considering at these points the fundamental solutions $\check{\Psi}(\vec{r}, \vec{r}_n)$ of wave equation (2) and allowing for the radiation condition, we construct the set of fundamental solutions $\{\check{\Psi}(\vec{r}, \vec{r}_n)\}_{n=1}^{\infty}$ with radiation centers at the specified points. Construct now the new set of vector-functions $\{\bar{\Phi}(\vec{r}^S, \vec{r}_n)\}_{n=1}^{\infty}$ defined on S such as each function of this set is derived from that of the preceding one by the relation

$$\bar{\Phi}(\vec{r}^S, \vec{r}_n) = \hat{W} \check{\Psi}(\vec{r}, \vec{r}_n) \Big|_{\vec{r}=\vec{r}^S}, \quad M(\vec{r}^S) \in S. \quad (4)$$

It can be shown [52], that for an arbitrary smooth surface S (in the Lyapunov sense) one can always find the auxiliary surface S' such that the constructed set of functions

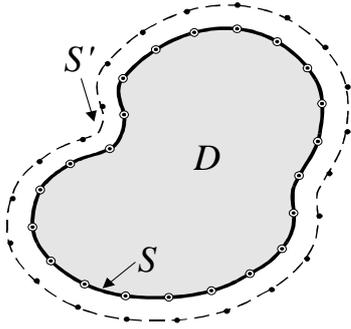


Fig.2. Problem and auxiliary geometry of domain D

$\{\bar{\Phi}(\vec{r}^S, \vec{r}_n)\}_{n=1}^{\infty}$ is complete and linearly independent on S in the functional space $L_2(S)$. In other words, if the auxiliary surface S' is chosen properly, any vector function being continuous on S can be expanded in terms of the first N functions of the set $\{\bar{\Phi}(\vec{r}^S, \vec{r}_n)\}_{n=1}^{\infty}$, and the expansion coefficients ensure obtaining a solution with any predesigned accuracy of the approximation as $N \rightarrow \infty$.

Applying the properties of the constructed set of functions to the right-hand side of equation (3), we can find the coefficients of best expansion of the function $\check{f}(\vec{r}^S)$ (in the sense of $L_2(S)$) in terms of the first N functions of the set

$$\{\bar{\Phi}(\vec{r}^S, \vec{r}_n)\}_{n=1}^{\infty}$$

$$\check{f}(\vec{r}^S) = \sum_{n=1}^N a_n \bar{\Phi}(\vec{r}^S, \vec{r}_n). \quad (5)$$

Then, the approximate solution to the considered boundary problem can be written as follows

$$\check{U}^{(N)}(\vec{r}) = \sum_{n=1}^N a_n \check{\Psi}(\vec{r}, \vec{r}_n), \quad \vec{r} \in D. \quad (6)$$

The expression (6) represents the expansion of an unknown field in terms of fundamental solutions of appropriate wave equation. The expansion coefficients a_n can be interpreted as the amplitudes of auxiliary sources, the fields of which are described by the fundamental solutions of wave equation (2) with radiation centers at the chosen points \vec{r}_n of the auxiliary surface.

It should be noted, that the properties of a specially chosen set of fundamental solutions $\{\check{\Psi}(\vec{r}, \vec{r}_n)\}_{n=1}^{\infty}$ guarantee the existence of coefficients $\{a_n\}_{n=1}^N$ providing the best in $L_2(S)$ mean-square approximation of the constructed solution (6) to the true solution $\check{U}(\vec{r})$, as well as the convergence of the approximate solution $\check{U}^{(N)}(\vec{r})$ to the exact solution $\check{U}(\vec{r})$ as $N \rightarrow \infty$.

3.2. Determining the unknown coefficients and estimating accuracy of the solution

To determine the unknown coefficients $\{a_n\}_{n=1}^N$, the optimal numerical method should be chosen, which would ensure obtaining the stable solution to the boundary problem (2)-(3) with predesigned accuracy and minimal computational resources. The accuracy of this solution can be estimated by the relative value of mean-square error (deviation) of fulfilment of the boundary conditions (3) on the surface S

$$\delta(N) = \left\| \hat{W}\tilde{U}^{(N)}(\vec{r}) - \tilde{f}(\vec{r}) \right\|_{L_2(S)} / \left\| \tilde{f}(\vec{r}) \right\|_{L_2(S)} \quad (7)$$

Among the methods for finding unknown coefficients a_n allowing, in principle, to reach any predesigned accuracy δ with increase of the number N of auxiliary sources, the following methods have been analyzed: the method of orthogonalization, the method of least squares, the method of moments, the collocation method and others. When comparing the possibilities of these methods, in addition to the requirement of providing the minimal deviation (7) of approximate solution (6) for the same N , the complexity of numerical realization (simplicity of matrix elements), as well as conditionality of the obtained algebraic system was taken into consideration.

The performed numerical experiments have revealed [8,9] the advantage of the collocation method for finding coefficients a_n because under the same conditions with this method maximal conditionality of the obtained algebraic matrix can be reached. Also, the coefficients a_n determined by the collocation method are close to the coefficients of the best mean-square approximation of the obtained solution to the true one. Finally, the collocation method provides, in contrast to well-established methods of orthogonalization and least squares, the most simplicity of matrix elements and, as a consequence, minimal computer resources.

When employing the collocation method, the boundary conditions (3) should be written at M points (collocation nodes) \vec{r}_m^S , $m=1, \dots, M$, where $M \geq N$. As the result, the problem is reduced, generally speaking, to the determination of pseudo-solutions of over-determined linear system of algebraic equations

$$\sum_{n=1}^N a_n \Phi_{\tau}(\vec{r}_m^S, \vec{r}_n) = f_{\tau}(\vec{r}_m^S), \quad m=1, \dots, M \quad (8)$$

where $\vec{\tau}$ is a unit vector tangential to the surface S at the point \vec{r}_m^S . Just as ordinary solutions appear when $M = N$, pseudo-solutions a_n also ensure the fulfillment of the boundary conditions (3) in average.

The obtained algebraic system is of the first kind, therefore it is, generally speaking, unstable with respect to the small perturbations of its right-hand side and requires additional examination for providing reasonable approximation of unknown solution to the true one. The latter depends on many factors, but above all, on the proper choice of auxiliary parameters which are the shape and dimensions of auxiliary surface, distribution of the nodes on the boundary and auxiliary surfaces, *etc.* The choice of auxiliary parameters, in turn, essentially depends upon the analytic properties of scattered field, namely, upon the character and location of singularities outside of the basic domain of continuously extended scattered field (we call them main singularities).

The wave fields singularities form the basis of the MAS, therefore let us consider them in more detail.

3.3. Wave field singularities from viewpoint of the MAS

The efficiency of the MAS essentially depends on whether the analytical properties of scattered field, namely, the character and location of its main singularities are properly taken into account. The examination of this point affects the proper choice of auxiliary surface, as well as the type and arrangement of the auxiliary sources. Moreover, ignoring this point leads to a weakening of convergence and even to a diverging of solution with increasing N . Therefore, analysis of the main singularities of scattered field is an essential part of the scheme to construct solution by means of the MAS.

As is well known, the singularities of wave field are the points in which the field becomes irregular (i.e. its continuity with derivatives are lost) or unbounded. In a homogeneous medium, the field's singularities coincide with the position points of the primary field sources. Similarly, in inhomogeneous media the secondary field sources (equivalent currents and charges) are introduced on the interfaces between the different media; they are responsible for the appearance of the scattered field and for breaking its regularity on the interfaces. These points are the singularities of the scattered field, and the full description of these singularities allows determination of the scattered field in the whole space. The implementation of such an approach makes up the basis of the method of singular integral equations.

The MAS assumes the more tidy analysis of analytical properties of wave fields examining the singularities of a continuously extended (together with its derivatives) scattered field $\tilde{U}(\vec{r})$ across the boundary S of its domain D . The basis for this analysis is the fact that any wave field, both scalar and vector, which is continuously extended to the whole space and satisfying the radiation condition at infinity, certainly has sources, i.e., irregular points (otherwise, it should be identically zero [66]). From this it follows, that if the wave field $\tilde{U}(\vec{r})$ is continuously extended across the boundary S of its domain D up to the boundary \tilde{S} of a new domain of its continuity, then we reach the set of singular points, or singularities of the continuously extended wave field. These singularities of the field we call the main singularities.

Each wave field possesses a unique set of the main singularities, which manifest themselves in the form of isolated singular points, open lines and surfaces. From the physical point of view, the main singularities can be interpreted as the images of the primary sources of the wave field (including, maybe, the secondary sources induced on other interfaces) in the boundary surface of scattering domain. Therefore, the character, location and weight of these singularities depend on both the shape of the scattering domain, and the characteristics of primary field sources. The complete knowledge of the main singularities of the wave field allows recovering of this field in its domain in a similar manner as with aid of usual singularities. However, in practice, it is impossible, as a rule, to determine all the characteristics of the main singularities. But to realize the MAS, the latter is not required: it is sufficient to determine only the location domain and the character of the main singularities.

The localization of the main singularities domain is necessary for the proper choice of the auxiliary surface: the latter should envelop their location domain, because it is needed for regularity of the expansion (6) outside the auxiliary surface. The various techniques have been developed to localize the main singularities domain [45,62]. Some of them are based on the analytical approaches, e.g., on searching of images of the given sources in the surface of the body [62] or on Fourier analysis of the directional pattern of the scattered field [45]. Techniques based on half-empirical approach, e.g., on the analysis of behavior of amplitudes and phases of the auxiliary sources while changing the shape of auxiliary surface, have been also developed [45]. However, in the general case of complicated shape of boundary surface and complex excitation kind, this problem can be solved only by numerical means.

Knowing the character of main singularities, in contrast to their location, is not necessary for implementing MAS, but it allows increase in efficiency of the MAS at the expense of modifying the choice of the type of auxiliary sources. The character of main singularities is determined by the behavior of the continuously extended scattered field in the vicinity of the singular points. Thus, the following kinds of main singularities are distinguished at the isolated singular points: the logarithmic singularity ($\propto \log |\vec{r}-r'|$), the pole of n -kind ($\propto |\vec{r}-r'|^{-n}$) and essentially singular point (the pole of ∞ -kind). The continuously distributed groups of singularities form the main singularities in the form of open lines and surfaces.

Thus, analysis of continuously extended scattered field (main singularities of the field) is an essential component for constructing algorithm of the MAS. Moreover, the continuous extension of scattered field is the physical essence of the MAS. In fact, the MAS uses the replacement of the main singularities of scattered field by the auxiliary ones, which are treated as the auxiliary sources of scattered field. From this it follows that to increase the efficiency of the MAS; the characteristics of the auxiliary sources should coincide with the characteristics of main singularities of the scattered field.

The problem of choosing auxiliary parameters is considered in the following section.

4. CHOICE OF AUXILIARY PARAMETERS

For numerical implementation of the MAS, it is necessary to properly choose the auxiliary parameters for ensuring the predesigned accuracy of approximation of boundary problem solution to the true one. The recommendations for choosing the main auxiliary parameters are given below.

4.1. Choice of auxiliary surface

The proper choice of the auxiliary surface plays the decisive role in numerical implementation of the MAS because this choice essentially affects the degree of conditionality, stability and the rate of convergence of the solution of algebraic system (8) with increase of the number N of auxiliary sources. Moreover, in case of improper choice of auxiliary surface, the computational process even diverges with increasing N .

To obtain a stable and well-conditional matrix, it is necessary to construct a system with dominating diagonal elements and maximal difference between the lines. Moreover, matrix elements should include the information concerning the scattering field's main singularities. Finally, the corresponding homogeneous algebraic system should not allow nontrivial solutions. Based on these requirements, we come to the following recommendations for choosing auxiliary surface:

1). The auxiliary surface S' should be equidistant from the boundary surface S , i.e., the distance d between the surfaces S and S' , measured along the normal, should be constant at each point. This ensures the approximate diagonality of the principal determinant of the system (8) and, therefore, the best conditionality and stability of the system (8) for the arbitrary N .

2). The surface S' should embrace the domain of location of the main singularities of scattered field. This follows from the specific character of the expansion (6), which describes, together with the unknown field in domain D , its continuous (with derivatives) extension up to the boundary S' of the extended domain D' . Due to the smoothness of the surface S , the condition stated above is equivalent to the requirement of absence of singularities in

intermediate domain confined by the surfaces S and S' . The violation of this condition results in the divergence of the computational process.

3). For a domain with complicated boundary S , the distance between the surfaces S and S' should satisfy the condition $d < R_{\min}^+$, where R_{\min}^+ is the minimal radius of positive curvature of the surface S . This condition, which is the consequence of the previous one, precludes the scattered field's main singularities connected with the geometry of the scatterer from being within the intermediate domain between the surfaces S and S' . It also indicates a way of investigating scattering problems upon bodies with non-smooth surfaces (edges), offering at edge points the finite rounding radius ρ_{\min}^+ and the choice of a surface S' according the requirement $d < \rho_{\min}^+$.

4). If the domain D is an outside one, the shape and dimensions of the auxiliary surface S' should be chosen in view of the resonant properties of the interior domain D' bounded by this surface. That means, the shape and dimensions of the domain D' should be such that the intrinsic frequencies of this domain do not coincide with the frequency of the primary field sources. Otherwise, the intrinsic field of the domain D' bounded by auxiliary surface could be added to the solution of scattering problem, because this field satisfies the corresponding boundary problem. In order to avoid this effect for scattering problems, one can use a solution trial and error by modifying auxiliary surfaces. On the other hand, the indicated effect allows creation of the effective algorithm for finding the intrinsic frequencies and fields of complicated regions [42-44] (the analysis of this phenomenon is performed in [8,9]).

The recommendations listed above set only upper limits for choosing the distance d between the boundary and auxiliary surfaces. The optimal distance d_{opt} between these surfaces is connected with the number N and distribution law for the nodes on the auxiliary surface, which, in turn, depend on the complexity of the described field and on the given accuracy of computations.

4.2. Distribution of nodes on the boundary and auxiliary surfaces

The number and distribution of the nodes $\{\vec{r}_n\}_{n=1}^N$ on the auxiliary surface (auxiliary sources) determine the degree of possible approach (in mean-square sense) of the sought solution of boundary problem to the true one (i.e. minimal possible deviation of the obtained solution). Approximation of the boundary surface by applying the collocation method leads to an additional degradation of solution accuracy. Therefore, the number and distribution of the nodes $\{\vec{r}_m^S\}_{m=1}^M$ on the boundary surface (collocation points) determine the realized accuracy (mean-square error δ) of this approximation.

When choosing the number N and distribution of auxiliary sources, one should proceed from the given accuracy of approximation and the complexity of a described field on the boundary surface S . It is obvious that the larger the characteristic dimensions of the surface S (relative perimeter $L_\lambda = L/\lambda$, where λ is the medium wavelength), the more complicated the field is to be described, and the more auxiliary sources must be used to describe this field under the given accuracy of computations. On the other hand, the choices of these parameters determine the optimal distance d_{opt} between the boundary and auxiliary surfaces. Indeed, decrease of the distance d between the surfaces leads to increasing conditionality of the algebraic matrix (8). However, non-uniformity of the described field along the boundary surface caused by the closeness of radiation centers of auxiliary sources is then increased. Therefore, to ensure the same computational accuracy, more terms in the expansion (8) for the

scattered field should be considered. Increase of d leads to the weakening of conditionality of the system matrix, but increasing uniformity of the described field along the boundary surface that needs a lower number N of auxiliary sources to describe the field with the same accuracy.

From the above considerations, it is clear that for a given solution accuracy δ in each specific case there exist an optimal relation between the relative number of auxiliary sources $n_\lambda = N / L_\lambda$ and the distance d_{opt} between the boundary and auxiliary surfaces. Also, the optimal distribution of auxiliary sources providing the minimal mean-square error should exist for each n_λ . In turn, the value of deviation of the obtained approximation depends on the character and coverage degree of the main singularities of scattered field, as well as on the relation between the numbers of collocation nodes M and auxiliary sources N .

The typical dependence of the solution convergence on the relative distance kd between the boundary and auxiliary surfaces (k is a wavelength in medium) is presented in Figs. 3 and 4 for different n_λ and various characters and locations of main singularities of the scattered field relative to the auxiliary surface. The above results are computed for the equidistant arrangement of the same number ($M=N$) of nodes on the boundary and auxiliary surfaces. Fig. 3 corresponds to the case, when the auxiliary surface passes across the main singularity of logarithmic (a) or simple pole (b) kind, and Fig. 4 to when the logarithmic main singularity is in the middle between the boundary and auxiliary surfaces.

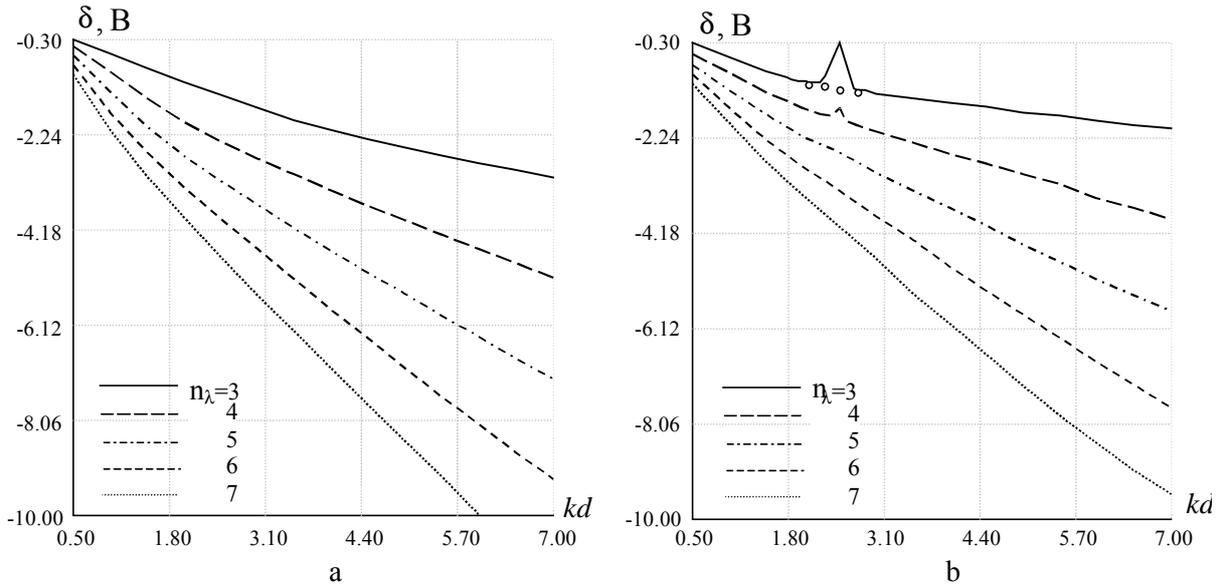


Fig. 3. The typical dependence of the solution convergence when the auxiliary surface passes across the main singularity of logarithmic (a) or simple pole (b) kind

Figs. 3,a,b show that if the auxiliary surface envelops the main singularities of the scattered field, a fast and monotone convergence of the results with increase of the relative number n_λ of auxiliary sources and relative distance kd is observed. In this case, only the finite number of computer digits restricts the achievable accuracy of computations, and to obtain the stable solution of the corresponding algebraic system (8), the following procedure of “soft” regularization by Tikhonov [67] is sufficient:

$$a_m \alpha + \sum_{n=1}^N a_n \Phi_\tau(\vec{r}_m^S, \vec{r}_n) = f_\tau(\vec{r}_m^S), \quad m=1, \dots, M. \quad (8a)$$

Here α is a regularizing complex parameter satisfying the condition $\alpha \ll \Phi_\tau(\vec{r}_m^S, \vec{r}_m)$ and providing the closeness of the solutions of the second kind equation (8a) to the first kind one

(8) (in our example, $|\alpha| \approx 10^{-14} \div 10^{-8}$). A more rapid (by order of magnitude) rate of the solution convergence in Fig. 3,a in comparison to that in Fig. 3,b is explained by presence of a stronger main singularity in the second case.

In Fig. 3,b one can also observe the above-mentioned sensitivity of the MAS to the resonance frequencies of the domain bounded by the auxiliary surface. This is shown in degradation of the solution convergence in vicinity of the resonance frequency ($kd \approx 2.45$). The observed resonance of auxiliary surface is easily eliminated by changing the shape of the auxiliary surface (markers in Fig. 3,b for $n_\lambda = 3$). With increasing N these resonances become so sharp ($n_\lambda = 5,6,7$), that their detection should the need arises, requires construction of special algorithms.

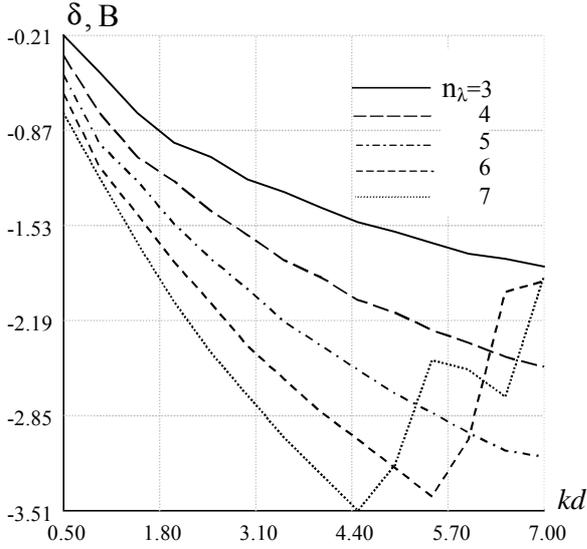


Fig. 4. Same as Fig. 3, but when the main singularity is between the boundary and auxiliary surfaces.

obtaining the minimal deviation. However, for creating the general computational algorithm valid for arbitrary geometry of the domain and incident wave, only such choice of distribution of nodes permits reasonable solution of the boundary problem. Therefore, to obtain the best-conditional system matrix, we recommend placing the auxiliary sources at the points of crossing auxiliary surface by the normals drawn from the collocation nodes on the boundary surface.

Should a priori information about the sought solution be known, the special algorithm of distribution of the nodes, taking into account specific features of the geometry and incident wave, can be implemented. In particular, for complicated surfaces one can consider a non-uniform and/or unequal number ($M > N$) of nodes on the boundary and auxiliary surfaces. As is stated by several authors [33,67], the case $M > N$ leads to the improvement of conditionality of the system matrix.

Thus, the proper choice of auxiliary surface and the number of nodes on the boundary and auxiliary surfaces allows a guaranteed solution of the boundary problem with predesigned accuracy.

The further increase of efficiency of the MAS is achieved by suitable selection of potential functions and auxiliary sources.

From Fig. 4 it follows that the presence of any main singularity of scattered field between the boundary and auxiliary surfaces, whatever weak it is, imposes a limit upon the increase of accuracy of solution with increasing kd . The full procedure of regularization [67] is also unable to increase the accuracy of the solution. Moreover, starting from some n_λ the results become diverge with increasing kd . As a result, the best accuracy obtained in Fig. 4 is about $\delta \approx 0.03\%$, while in Fig. 3a,b where the auxiliary surface envelops the main singularities of the field, it is about $\delta \approx 10^{-10}\%$.

The uniform distribution of the same number ($M=N$) nodes on the boundary and auxiliary surfaces is not always optimal for

4.3. Selection of the type of potential functions and auxiliary sources

By potential function, as mentioned above, we imply the function identically defining vectors of the electromagnetic field. As such a function, e.g., the following quantities might be considered: electric and magnetic electrodynamic vector potentials $\vec{A}^{e,m}$, Debye scalar potentials ${}^{e,m}\Pi$ [68], Hertz vectors $\vec{Z}^{e,m}$ [69], spinor dyad of Hertz potential \vec{Z} , spinor dyad of electromagnetic field \vec{F} [70] and, finally, arbitrary field vectors $\{\vec{E}, \vec{H}, \vec{D}, \vec{B}\}$.

Each potential function satisfies its own wave equation and is related to the field vectors in a certain way. In turn, the specific form of equations depend on the constitutive relations of the medium, therefore, each potential function is characterized by own wave operator \hat{L} and own relations with field vectors.

Thus, when choosing as a potential function the electric Hertz vector ($\vec{U} \equiv \vec{Z}^e$), the wave operator in isotropic magneto-dielectric is as follows

$$\hat{L} = \vec{\nabla}^2 + k^2 \quad (9a)$$

where $\vec{\nabla}$ is a Hamilton operator, and $k = \omega\sqrt{\epsilon\mu}$ is a wavenumber in dielectric. The unknown field vectors are then expressed via chosen potential function in a certain way [69]

$$\vec{E} = \vec{\nabla} \times \vec{\nabla} \times \vec{U}, \quad \vec{H} = -i\omega\epsilon \vec{\nabla} \times \vec{U} \quad (10a)$$

When choosing as potential function any field vector, e.g. $\vec{U} \equiv \vec{E}$, the wave operator, as a rule, has a more intricate form, however the relations for field vectors in this case appear to be simpler. Thus, for isotropic magnetodielectric we obtain

$$\hat{L} = \vec{\nabla} \times \vec{\nabla} \times \vec{I} - k^2 \vec{I} \quad (9b)$$

$$\vec{E} = \vec{U}, \quad \vec{H} = \frac{1}{i\omega\mu} \vec{\nabla} \times \vec{U} \quad (10b)$$

In complex media such as chiral and biisotropic which distinguish the direction of polarization rotation (right-handed r and left-handed ℓ) the role of Hertz vector plays the spinor dyad of Hertz potentials $\vec{U} \equiv \vec{Z} = \begin{Bmatrix} \vec{Z}^r \\ \vec{Z}^\ell \end{Bmatrix}$ satisfying the wave equation (2) with operator

$$\hat{L} = \vec{\nabla}^2 + k^2 \quad (9c)$$

where $k = \begin{pmatrix} k_r & 0 \\ 0 & -k_\ell \end{pmatrix}$ is a wavenumber matrix, the elements of which depend on the parameters of medium by intricate manner. The field vectors are found to be connected with the potential function via relations

$$\vec{E} = \vec{\chi} \hat{\gamma} \vec{U}, \quad \vec{H} = \vec{\xi} \hat{\gamma} \vec{U} \quad (10c)$$

where $\vec{\chi}$ and $\vec{\xi}$ are two-element rows of parameters, $\hat{\gamma} = \begin{pmatrix} \hat{\gamma}^r & 0 \\ 0 & \hat{\gamma}^\ell \end{pmatrix}$ is a matrix differential operator with elements $\hat{\gamma}^{r,\ell} = \vec{\nabla} \times \vec{I} \pm k_{r,\ell}^{-1} \vec{\nabla} \vec{\nabla} \pm k_{r,\ell} \vec{I}$, and \vec{I} is a unit dyadic [64,65].

For description of various media, it is convenient to introduce as a potential function, the spinor dyad of electromagnetic field $\vec{U} \equiv \vec{F} = \begin{Bmatrix} \vec{F}^r \\ \vec{F}^\ell \end{Bmatrix}$ satisfying the wave equation (2) with operator

$$L = \vec{\nabla} \times \vec{I} - k \vec{I} \quad (9d)$$

In view of relation $\vec{F} = \hat{\gamma} \vec{Z}$, the electromagnetic field vectors in this case are related to the potential function by extremely simple expressions

$$\vec{E} = \bar{\chi} \vec{U}, \quad \vec{H} = \bar{\xi} \vec{U} \quad (10d)$$

Thus, any choice of the potential function allows mathematical formulation of the boundary problem and determination of the electromagnetic field vectors. However, the choice of the potential function influences the complexity of wave operator, as well as the relations connecting the field vectors with the potential function. Therefore, the ability to determine the fundamental solutions of appropriate wave equation plays the conclusive role when choosing the potential function. This is obligatory for implementing the scheme of the MAS (this is the reason why a conventional MAS is also named as the method of expansion by fundamental solutions [8,9]).

To effectively implement the MAS, it is also necessary to properly choose the type of auxiliary sources. However, the fields of the same auxiliary sources can be described by different potential functions. For example, the field of elementary auxiliary sources (elementary electric and magnetic currents) is described both with the help of fundamental solutions

$$\vec{\Psi}(\vec{r} - \vec{r}') = \frac{1}{4\pi} \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \vec{\tau} \quad (11a)$$

of wave equation with Helmholtz operator (9a), and with the help of fundamental solutions

$$\vec{\Psi}(\vec{r} - \vec{r}') = \vec{\nabla} \times \vec{\nabla} \times \frac{1}{4\pi} \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \vec{\tau} \quad (11b)$$

of wave equation with Helmholtz-like operator (9b). In these expressions, \vec{r}' denotes the source point, and $\vec{\tau}$ a unit vector of direction of source current at this point. However, (11a) corresponds to the choice of Hertz vector as the potential function, while (11b) to that of the electric field vector.

Similarly, both the fundamental solutions

$$\vec{\Psi}(\vec{r} - \vec{r}') = G(\vec{r} - \vec{r}') \vec{\tau} \quad (11c)$$

of wave equation with operator (9c) for spinor dyad of Hertz potentials, and fundamental solutions

$$\vec{\Psi}(\vec{r} - \vec{r}') = \hat{\gamma} G(\vec{r} - \vec{r}') \vec{\tau} \quad (11d)$$

of wave equation with operator (9d) for spinor dyad of electromagnetic field describe the field of auxiliary sources in the form of spin-vector dyad of elementary currents $\vec{\tau} = \left\{ \begin{matrix} \vec{\tau}^r \\ \vec{\tau}^\ell \end{matrix} \right\}$ which are rotated in opposite directions (clockwise and counter-clockwise). Here

$$G = \begin{pmatrix} G^r & 0 \\ 0 & G^\ell \end{pmatrix}, \quad G^{r,\ell}(\vec{r} - \vec{r}') = \frac{1}{4\pi} \frac{e^{ik_{r,\ell}|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \quad (12)$$

is the matrix of fundamental solutions of scalar Helmholtz wave equation.

When solving specific problems especially with some kind of symmetry, along with elementary auxiliary sources, compound (combined and integrated) auxiliary sources can be used. The fields of these sources can be represented, accordingly, as the sums and integrals of the fields of elementary sources. The combined sources describe the fields of finite set of elementary sources, e.g., the combinations of electric and magnetic dipoles creating the fields of heterogeneous polarization. The integrated sources describe the fields of continuous aggregates of elementary sources, e.g., the sources continuously distributed along the ring, disk or other smooth ring-supported surface. The field of latter sources can be describing by Deschamps functions [71], i.e., by cylindrical $H_0^{(1)}(x)$ or spherical $h_0^{(1)}(x)$ Hankel functions of complex argument. The fields of metaharmonic functions (Hankel functions of higher

orders) can also be referred to as integrated sources (they are convenient for description of the fields of round-cylindrical and spherical surfaces).

The choice of the type of auxiliary sources is determined by the specific character of the problem to be solved, but above all, by the character of the main singularities of scattered field. To achieve a higher efficiency of the MAS, the type and disposition of the auxiliary sources should coincide with the character and location of the main singularities of scattered field. Thus, for logarithmic main singularity, the auxiliary sources described by Hankel functions of zero order $H_0^{(1)}(k|r-r'|)$ should be chosen, while for the pole of n -order, cylindrical and spherical Hankel functions of n -order should be applied. For the main singularities continuously distributed on the open lines and surfaces, the auxiliary sources described by Hankel functions of complex argument (Deschamps functions) can be used.

Application of the compound auxiliary sources enables a solution to the initial problem with minimal effort that leads finally to maximal efficiency of the employed method. However, for lack of additional information concerning the character and location of the main singularities, the proper use of elementary auxiliary sources always allows the boundary problem to be solved.

5. APPLICATION TO PARTICULAR PROBLEMS

In the sections above, construction of the MAS solution was clarified and recommendations for choosing the auxiliary parameters to obtain the optimal solution were given. In this section, we will consider the application of the MAS for solving 2D and 3D scattering problems upon single and set of bodies made of various materials: isotropic, anisotropic and chiral.

The geometry of the problems to be studied is as follows. As 2D, the cylindrical bodies with elements along the z axis and excitation in a transverse plane are considered, while as 3D, the bodies of rotation excited along the z axis of rotation by plane waves are considered. Because of the symmetry of these problems, in the 2D case, the boundary conditions can be written only on the contour of cylinder cross-section, while in 3D case the initial problem can be transformed into the problem of two circular polarized waves excitation, and the boundary conditions can be written on the semi-contour of axial section of the body.

In the calculations performed, the numbers of auxiliary sources are equal to the number of collocation nodes ($N=M$).

5.1. Electromagnetic scattering upon the anisotropic bodies

Anisotropic materials are widely used in microwave engineering to develop various electronic devices. Therefore, the scattering problems upon the bodies of anisotropic materials are of great interest. We consider here one of the problems connected with forming of narrow-directional radiation with aid of a magneto-dielectric anisotropic cylinder.

The elementary calculations show that an isotropic magneto-dielectric spheroid with semi-axis a , b , c (or elliptical cylinder with semi-axis a , b) forms a narrow-directional radiation, if only a point (linear) source is placed in a focus, and the semi-axis ratio and material parameters are related as follows: $a/b = n/\sqrt{n^2-1}$, where $n = \sqrt{\epsilon_r \mu_r}$ is a refractive index of medium. The focal distance of such a body is $f = \sqrt{a^2 - b^2}$, where a and $b=c < a$ are the semi-major and semi-minor axes.

This idea can be used for implementing the same process with aid of spherical or round-cylindrical anisotropic body. Consider here 2D case with z-axis oriented along the element of cylinder.

If we introduce along with reference frame (x, y, z) the new reference frame (x', y', z') related new to the old one by a linear transformation $(x' = x\sqrt{\mu_{yr}}; y' = y\sqrt{\mu_{xr}}; z' = z)$, then the medium is isotropic in a new frame with the refractive index $n' = \sqrt{\epsilon_{zr}}$. Also field components in the new frame satisfy the usual isotropic Helmholtz equation with fundamental solutions

$$\Psi(x', y', z') = H_0^{(1)}(k_0 \sqrt{\epsilon_{zr}} \sqrt{(x' - x'_0)^2 + (y' - y'_0)^2}) \quad (13)$$

with centers at the points (x'_0, y'_0, z') . Here k_0 is free space wavenumber.

Expression (13) is rewritten in an old reference frame as follows

$$\Psi(x, y, z) = H_0^{(1)}(k_0 \sqrt{\epsilon_{zr}} \sqrt{\mu_{yr}(x - x_0)^2 + \mu_{xr}(y - y_0)^2}) \quad (14)$$

The circular region $x^2 + y^2 = \frac{d^2}{4}$ in old reference frame (d is a cylinder diameter) corresponds in a new one to an elliptical region $\frac{x'^2}{\mu_{yr}} + \frac{y'^2}{\mu_{xr}} = \frac{d^2}{4}$ with semi-axes $a' = \frac{d}{2} \sqrt{\mu_{yr}}$, $b' = \frac{d}{2} \sqrt{\mu_{xr}}$ and focal distance $f' = \frac{d}{2} \sqrt{\mu_{yr} - \mu_{xr}}$. Therefore, if we place a linear source into an imaginary focus $(\pm \frac{d}{2} \sqrt{1 - \mu_{yr} / \mu_{xr}}, 0)$ of a circular anisotropic cylinder, then the radiation will be narrow-directional along the focal distance, if only the components of dielectric and magnetic tensors satisfy the condition $\epsilon_{zr} = \mu_{yr} / (\mu_{yr} - \mu_{xr})$, $\mu_{yr} > \mu_{xr}$ being the consequence of $a' / b' = n' / \sqrt{n'^2 - 1}$.

Fig. 5,a,b shows a distribution of radiating energy density produced by anisotropic circular cylinder outside (a) and inside (b) the cylinder drawn after solution of the boundary problem. The material parameters of the problem are as follows: $\epsilon_{zr} = 1.6667$, $\mu_{xr} = 1.0$, $\mu_{yr} = 1.5625$, $k_0 d = 350$. Fig. 5a shows a narrow-directional beam behind the cylinder and an intricate interference picture around the cylinder. Fig. 5,b reveals clearly a location of a real source to the left of the center of cylinder, an imaginary focus to the right of the cylinder, also forming of a beam structure inside the cylinder.

Thus, knowing the medium type and its material parameters, the MAS algorithm can be used to solve the scattering problem and to analyze the desired physical problem.

5.2. Electromagnetic scattering upon the chiral bodies

In recent years, new complex media have become of interest due to their special properties and potential applications [64,65,72-77]. Among them, the so-called chiral medium is of greatest interest. As was mentioned above, chiral medium can be described by three scalar material parameters, i.e., ϵ , μ and $\alpha = \beta$. The most significant property of this medium is its handedness, i.e. the sensitivity with respect to the rotation direction of polarization plane of transmitted wave (clockwise or counter-clockwise). As a result, chiral medium is characterized by dyad of wavenumbers

$$k_{r,\ell} = k [(1 + \eta^2 \alpha^2)^{1/2} \pm \eta \alpha], \quad (15)$$

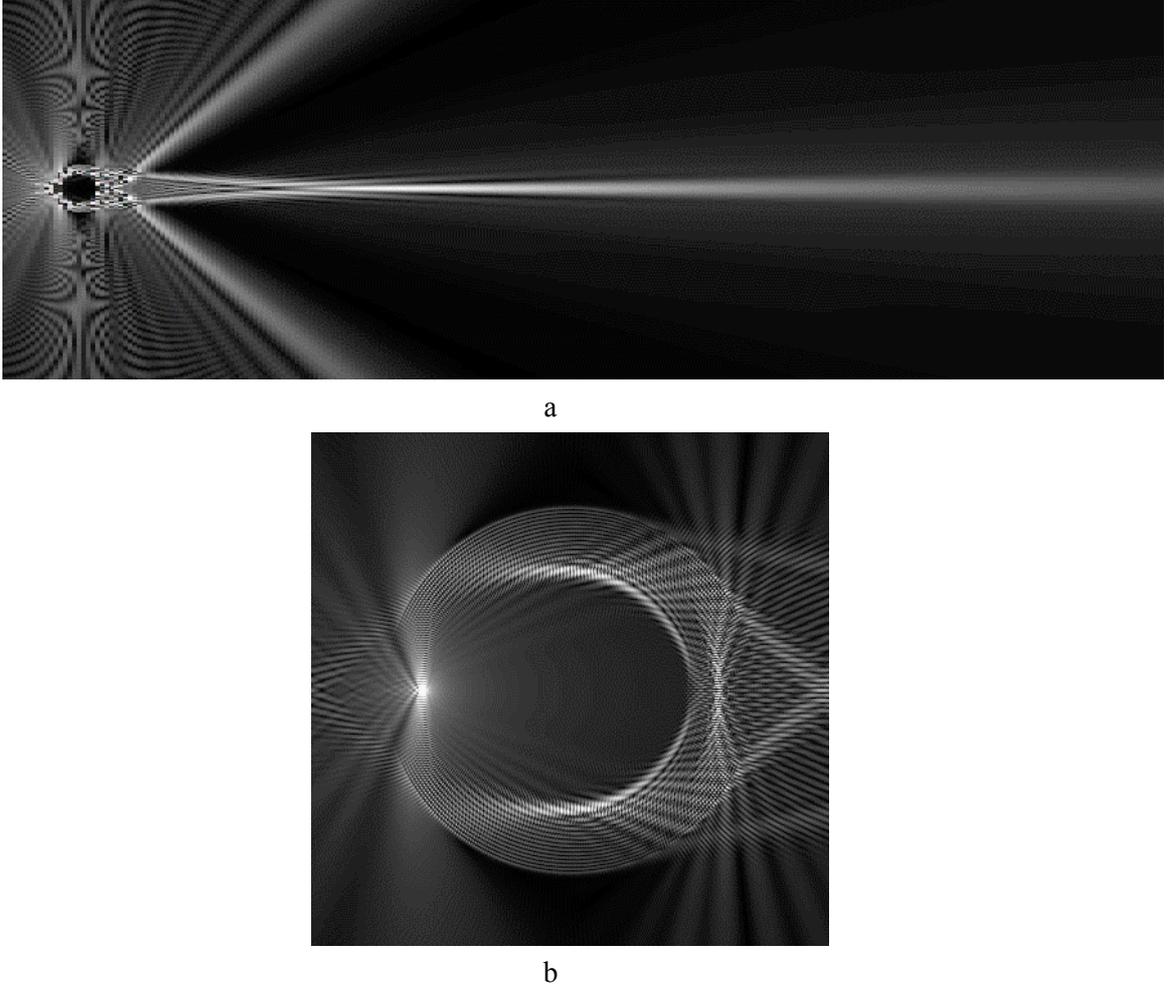


Fig. 5. Distribution of the total energy density outside (a) and inside (b) anisotropic circular cylinder.

where $k = \omega\sqrt{\epsilon\mu}$ and $\eta = \sqrt{\mu/\epsilon}$ are not the physically relevant wavenumber and wave impedance. To describe the fields in chiral medium fully, it is also necessary to determine the rows of parameters $\bar{\chi}$ and $\bar{\xi}$ incoming in expressions (10b) and (10d)

$$\bar{\chi} = (\frac{1}{2}i\eta_c, -\frac{1}{2}i\eta_c), \quad \bar{\xi} = (\frac{1}{2}, \frac{1}{2}), \quad (16)$$

with $\eta_c = \eta/(1 + \eta^2\alpha^2)$ being the wave impedance of chiral medium. Then, (10c) and (10d) represents the decompositions of electromagnetic field by right- and left-polarizations.

Below, some numerical results for 2D and 3D scattering upon the single chiral bodies both of canonical and complicated shape are presented and analyzed. The distinguishing feature of this case is the appearance of cross-polarized fields in external medium along with co-polarized ones because of coupling between the transverse polarizations in chiral medium.

To verify the MAS solution in 2-D case, Fig. 6 shows a comparison of co-polarized and cross-polarized differential scattering cross-sections

$$\sigma(\theta, \theta_0) = \lim_{r \rightarrow \infty} 2\pi r |\vec{H}|^2 / |\vec{H}_0|^2 \quad (17)$$

of a circular lossy chiral cylinder of diameter d illuminated by a TE to z polarized plane wave ($H_z^0 = e^{ik_0x}$) with those obtained by eigenfunction method [72] and volume integral equations approach [74]. The problem is characterized by the following parameters: $d=0.3$ m,

$k_0d=1.8863$, $\epsilon_r = 3.0 + i0.15$, $\mu_r = 2.0 + i0.10$, $\alpha = \beta = 0.002$, $\theta_0 = 180^\circ$. Hereinafter, k_0 is a free space wavenumber, $\epsilon_r = \epsilon/\epsilon_0$, $\mu_r = \mu/\mu_0$, and θ and θ_0 are the observation and source polar angles.

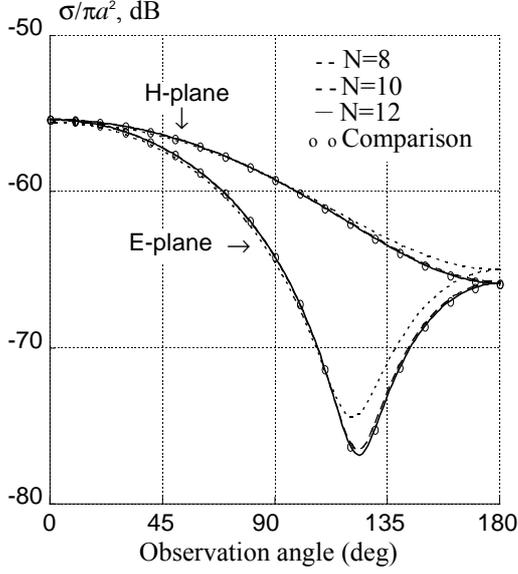


Fig. 6. Comparison of the scattering cross-sections of a circular lossy chiral cylinder with those obtained in [72] and [74]

We are comparing here 2 cases for $N=12$ and 14 auxiliary sources (collocation nodes).

The inset in Fig. 6 shows a quick convergence of MAS results with increasing N , so that for $N=14$ they are in excellent agreement with exact ones quoted from [72]. Moreover, the MAS results for $N=14$ are significantly more accurate than those obtained by volume integral equations approach for 763 cells [74] (the latter ones are close to the MAS results for $N=12$). It should also be noted, that deviation of the MAS results for $N=14$ are less than 0.1%. These reasons confirm the validity of the proposed method in chiral case and its significant advantage in comparison to well-known ones.

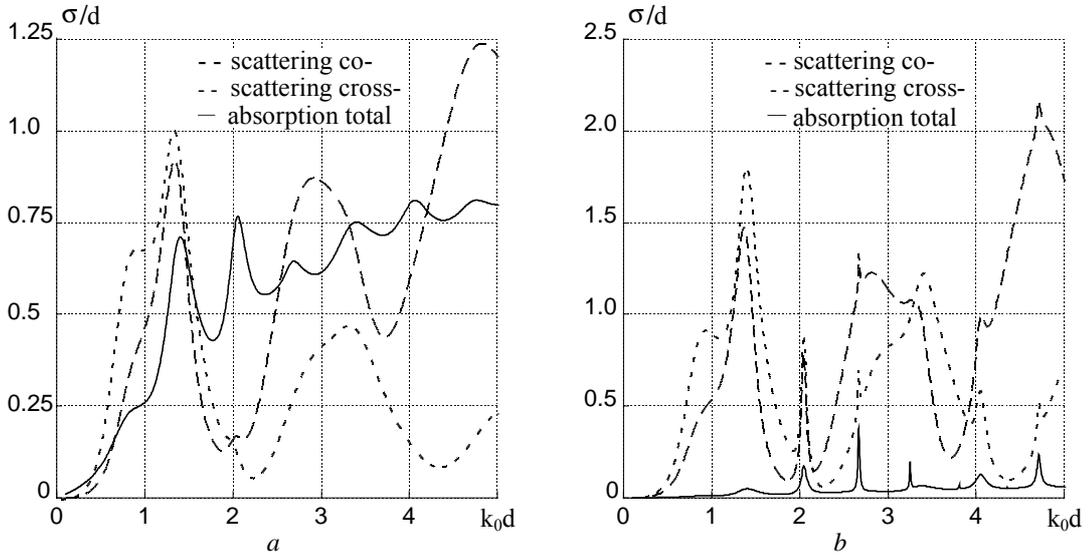


Fig. 7. Scattering and absorption cross-sections of a lossy chiral cylinder for large (a) and less (b) absorption.

In order to study electrodynamic properties of 2D chiral bodies in a wide frequency range, Fig. 7,a shows the normalized co-polarized and cross-polarized scattering cross-sections

$$\sigma = \lim_{r \rightarrow \infty} \text{Re} \frac{\int_L \{ \vec{E}(\vec{r}) \times \vec{H}^*(\vec{r}) \} \cdot \vec{n}(\vec{r}) d\ell}{|\vec{E}_0 \times \vec{H}_0^*|}, \quad (16)$$

and the total absorption cross-section

$$\sigma_{abs} = \text{Re} \frac{\int_L \{ [\vec{E} + \vec{E}_0] \times [\vec{H} + \vec{H}_0]^* \} \cdot \vec{n}(\vec{r}) d\ell}{|\vec{E}_0 \times \vec{H}_0^*|}, \quad (17)$$

for a TE to z plane wave incident upon a lossy chiral cylinder with material parameters of Fig. 6 versus the non-dimensional wave parameter $k_0 d$. To evaluate the influence of absorption upon the scattering plots, Fig. 7,b shows the same dependences for considerably lesser values of dielectric and magnetic losses ($\epsilon_r = 3.0 + i0.006$, $\mu_r = 2.0 + i0.004$).

From analysis of Fig. 7,a,b we gather that scattering and absorption plots of the chiral cylinder are strongly modified with increasing frequency of incident wave. Besides, the cross-polarized scattering cross-section of chiral cylinder is as large as, or sometimes larger, than the co-polarized one. These plots also show the absorption maximums corresponding to the resonant frequencies of oscillations inside the chiral cylinder.

The presence of losses essentially affects the course of scattering and absorption plots. Thus, decrease of losses in Fig. 7,b in comparison to Fig. 7,a leads to a significant increase of scattering level, change of the scattering structure and redistribution of energy between the polarizations. This process intensifies with increasing wave parameter $k_0 d$, because of more and more oscillations arising inside the cylinder. Decrease of the losses especially manifests itself in increasing the quality of resonances and formation of sharp peaks on the scattering and absorption plots.

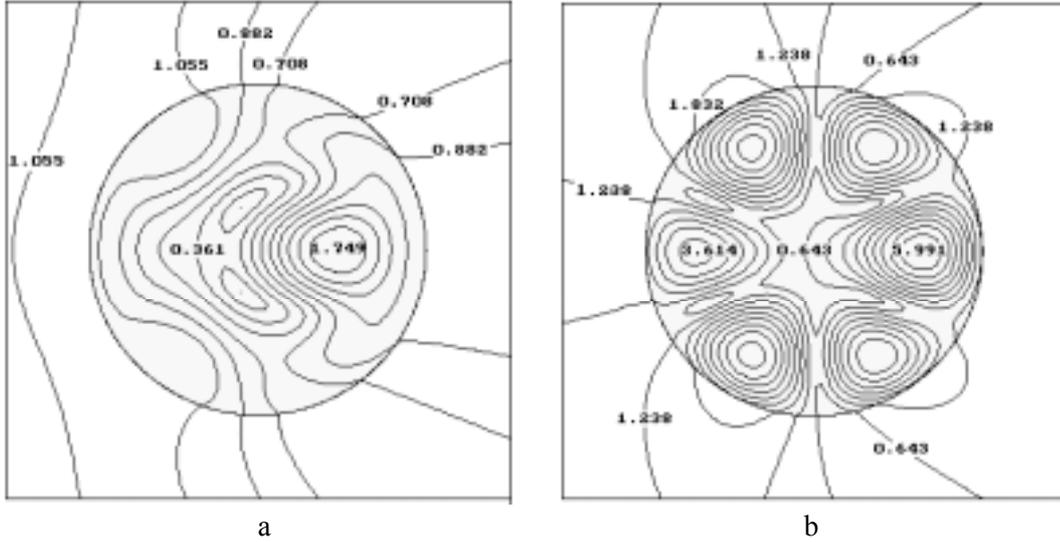


Fig. 8. Distribution of the co-polarized component of near magnetic field in a maximum of absorption plot in Fig. 7 for larger (a) and lesser (b) absorption

To study the structure of eigen-oscillations, Fig. 8,a,b depicts the normalized co-polarized component H_z of near magnetic field for one of the maxima in absorption plot for larger (a) and lesser (b) absorption. One can clearly see in Fig. 8,b the oscillations of whispering gallery for $k_{res} d = 2.6686$ with 3 total vibrations along the perimeter, one vibration along the cylinder radius and maximum of the internal field magnitude of 5.99 regarding the

incident field magnitude. Fig. 8,a shows the shift of the resonant frequencies ($k_{res}d=2.692$), decrease of the internal field magnitude maxima (1.749) and destruction of resonances because of decay of the oscillations quality for the larger losses.

To verify the MAS solution in 3D case, Fig. 9 presents the comparison between the results for the normalized total scattering cross-section

$$\frac{\sigma(\theta, \varphi)}{\pi a^2} = \lim_{r \rightarrow \infty} \operatorname{Re} \frac{1}{\pi a^2} \frac{\int_s \{\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})\} \cdot \vec{n}(\vec{r}) dS}{|\vec{E}_0 \times \vec{H}_0^*|}, \quad (18)$$

for a plane wave incidence upon the chiral sphere of radius a versus the zenithal angle θ in the E- and H-planes ($\varphi = 0$ and $\varphi = \pi/2$ accordingly), calculated by the MAS and the method

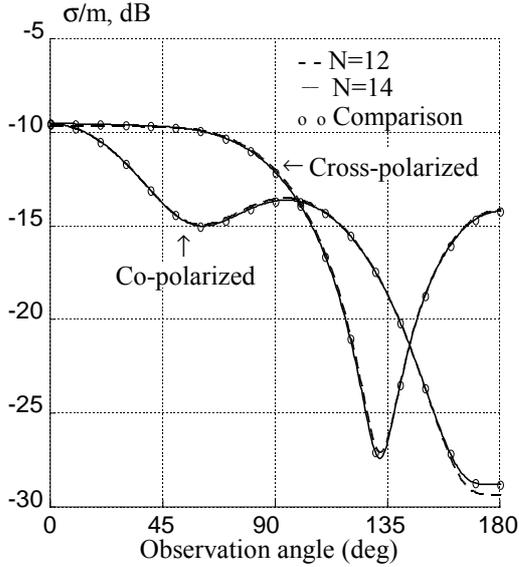


Fig. 9. Comparison of the total scattering cross-section of a chiral sphere with that quoted from [75].

of integral equations [75]. We gather from Fig. 9 that the quick convergence of algorithm with growing N is achieved, so that for $N=10$ the deviation of the results is about 1%, and for $N=12$ the results are indistinguishable from the accurate ones and are in excellent agreement with those quoted from [75].

To study the scattering properties of chiral bodies in a wide frequency range and to compare them to achiral case, Fig. 10 shows the normalized total scattering cross-section of chiral and magneto-dielectric spheres versus the wave parameter k_0d . The material parameters of chiral spheres are those as in Fig. 9, and for magneto-dielectric are the same except $\alpha=0$. Besides, a similar curve for chiral spheres obtained by the formulae of [75] is also depicted in Fig. 10 for the purpose of comparison to MAS results.

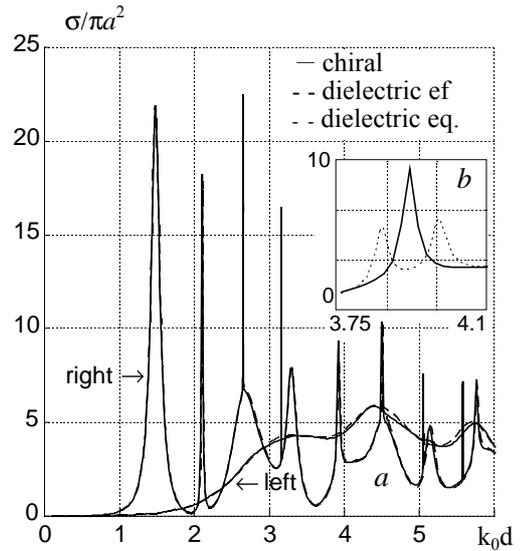


Fig. 11. Same as in Fig. 10, but for chiral, equivalent and effective dielectric spheres exposed to the circular polarized waves.

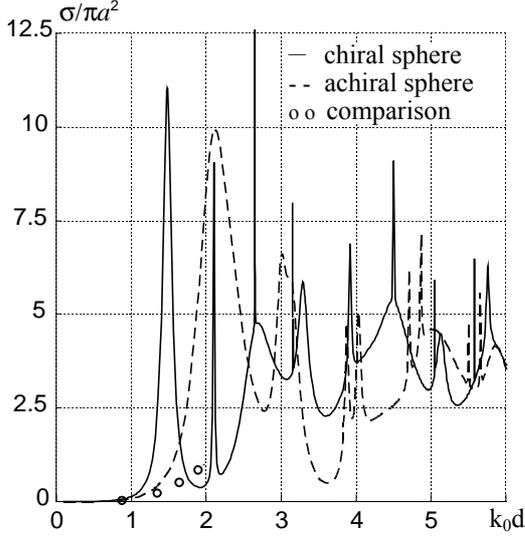


Fig. 10. Total scattering cross-sections of the chiral and achiral spheres exposed to a plane wave versus the wave parameter.

when illuminating the magneto-dielectric

$$\bar{\epsilon}_{ef}^{r,\ell} = \bar{\mu}_{ef}^{r,\ell} = \sqrt{\epsilon_{eq}^{r,\ell} \mu_{eq}^{r,\ell}}$$

spheres with so-called effective parameters are also presented in Fig. 11a. Here $\epsilon_{eq}^{r,\ell}$ and $\mu_{eq}^{r,\ell}$ are well-known equivalent parameters of chiral medium introduced as the proportionality factors between the right- and left-polarized wave contributions ($\vec{D}^{r,\ell} = \epsilon_{eq}^{r,\ell} \vec{E}^{r,\ell}$, $\vec{B}^{r,\ell} = \mu_{eq}^{r,\ell} \vec{H}^{r,\ell}$) [65]. Fig. 11,b presents a comparison of the results for chiral and equivalent magneto-dielectric spheres.

Upon inspection of Fig. 11,a we conclude that chiral body, in contrast to the achiral one, exhibits the sensitivity with respect to the rotation direction of incident wave polarization plane. Besides, each resonance is associated with a certain rotation direction. And what is more important, the chiral body behaves as a magneto-dielectric one with corresponding effective parameters (the plots for chiral and effective magneto-dielectric are almost indistinguishable in Fig. 11,a). The distinction of effective magneto-dielectric from equivalent one is that the former is best matched to free space by wave impedance. Thus, the effective magneto-dielectric, unlike the equivalent one, does not cause the resonance splitting resulting in appearing thin structure in scattering plots (see Fig. 11,b).

Fig. 10 reveals that for smaller k_0d the quasi-static model employed in [75] is true, and the scattering cross-section of chiral sphere satisfies the Rayleigh law of scattering. However, with growing k_0d , a strong difference between the compared results arises, and to obtain the true results, application of a dynamical model is necessary. In the case of further increasing of k_0d , resonance effects similar to those of the 2D case appear. Comparison between the chiral and achiral results shows, that the beginning of resonance domain is shifted to the left with growing chirality.

To analyze the scattering plots of Fig. 10 for chiral spheres in more detail, Fig. 11,a presents the same plots for right- and left-polarized plane waves incidences. Similar plots

are presented in Fig. 11,b. Similar plots are presented in Fig. 11,c. Similar plots are presented in Fig. 11,d.

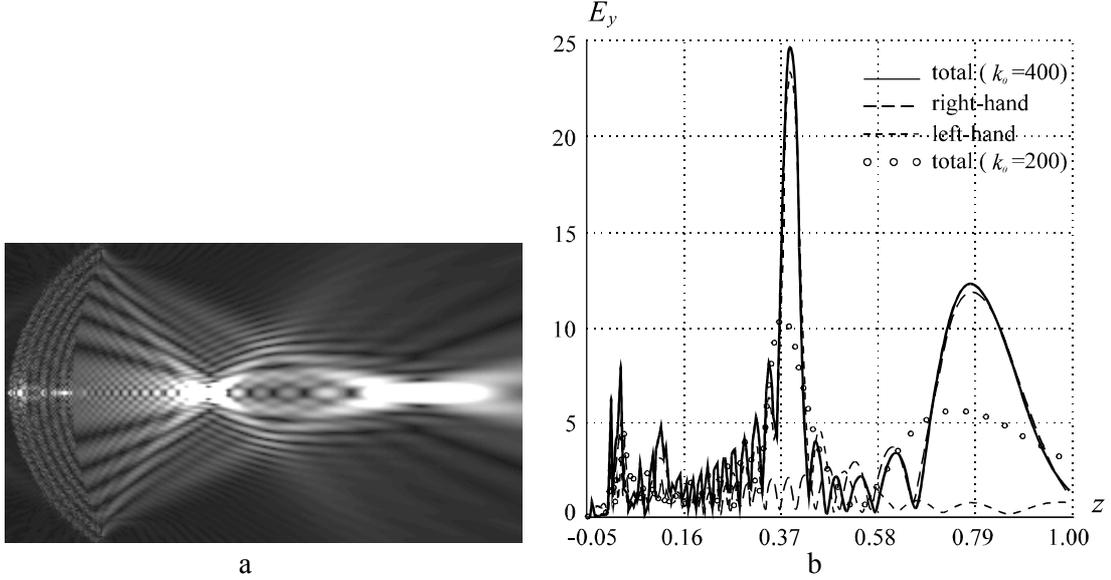


Fig 12. Distribution of cross component of electric field inside and outside of a special shape chirolens: a - the summarized polarization in axial cross-section, b - the summarized polarization and circular contributions along the axial line.

To study the behavior of chiral bodies in high-frequency range, Fig. 12 shows the focusing process performed by a special shape non-aberrational chirolens. The 3D lens with height $d=0.6$, thickness $t=0.125$ and material parameters $\epsilon_r = 3.0$, $\mu_r = 1.389$ and $\alpha = \beta = 0.3/120\pi$ is exposed to a plane wave with $k_0=400$. Fig. 12,a shows a distribution of cross-polarized component of the total electric field in axial cross-section of the lens. The same field distribution along the axis of the lens, as well as the distributions for the right-hand and left-hand circular waves contributions are depicted in Fig. 12,b.

From analysis of Fig. 12,a we gather, that the chirolens forms the two bright focal spots separated in space. Fig. 12,b shows, that each of the focal spots in Fig. 12,a is created by the single handedness of incident wave. Thus, the chirolens exhibits the property of space separation of waves with opposite handedness. Comparison of field amplitudes in the centers of the focal spots with those calculated for a smaller wavenumber ($k_0=200$) [62] shows that the larger wavenumber of incident wave, the stronger focusing effect. It should be noted, that although the relative dimensions of the lens correspond to the low limit of quasi-optics, Fig. 12,a shows all the characteristic properties of wave propagation and focusing in optical band.

5.3. Electromagnetic scattering upon the sets of bodies

Let us consider, finally, the results of computer modeling of the general scattering problem of Fig. 1. This is the problem of scattering of electromagnetic waves generated by given electromagnetic sources upon the set of bodies of complicated shape and filling. It should be noted, that problems of such kind belong to the most important boundary problems, from the practical point of view. On the other hand, these problems are also the most intricate because of interference between the fields scattered by every body. However, if the method employed provides the predesigned accuracy of calculations, the possibility to solve them depends only on the computer resources.

The geometry of 2D scattering problem to be considered is shown in Fig. 1. The set of bodies consists of an isotropic triangular shaped dielectric cylinder with material parameters $\epsilon_{r,1} = 3.0 + i0.001$, $\mu_{r,1} = 1.0 + i0.0$ (domain D_1), an elliptically shaped ($b/a=1/3$) real conductor

with the parameters $\varepsilon_{r2} = 3.0 + i3.0$, $\mu_{r2} = 1.0 + i0.0$ (domain D_2), an anisotropic magneto-dielectric cylinder with oval Kassini cross-section with the parameters $\varepsilon_{zr3} = 3.0 + i0.0$, $\mu_{xr3} = 1.5 + i0.01$, $\mu_{yr3} = 2.0 + i0.01$ (domain D_3) and an ideally conductive screen (domain D_4). This set of bodies is exposed to a TM to z polarized narrow-directional beam described by the Deschamps function (the function of Hankel of the complex argument)

$$E_z^0 = J_0 H_0^{(1)}(k_0 \sqrt{(x-x_0)^2 + (y-y_0)^2}) \quad (19)$$

Here $x_0 = 5.0 - i160.0$ m, $y_0 = 3.0 + i160.0$ m, $k_0 = 80 \text{ m}^{-1}$ is the free space wavenumber, J_0 is a coefficient providing the unit amplitude of incident wave at the origin of the reference frame.

Fig. 13 shows the distribution of the amplitudes of the total near electric field inside and outside the bodies. The dimensions of the depicted domain correspond to $100 \lambda \times 100 \lambda$, beam width is about 15λ , and dimensions of the bodies are about 20λ (here λ is the wavelength in free space). One can clearly see in Fig. 13 the interference structure of the field inside the magneto-dielectric and in free space, focusing process inside the anisotropic body and the rapid attenuation of the field inside the conductor. It should be emphasized that only sufficient accuracy of calculations (about 0.03%) allows a detailed description of the field to be obtained.

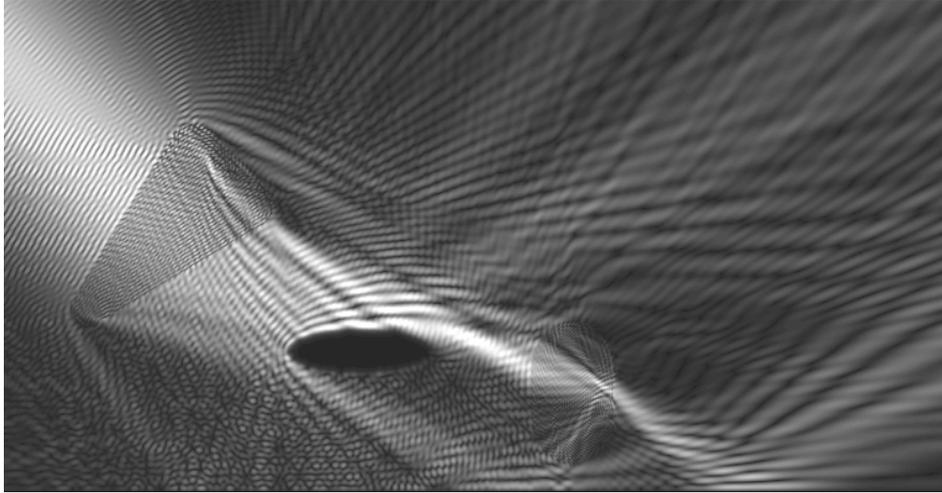


Fig. 13. Distribution of the co-polarized component of electric field inside and outside the set of complicated shape bodies of Fig. 1 with various material properties: dielectric (1), real conductor (2), anisotropic magneto-dielectric cylinder (3) and screen (4).

Thus, the proper use of the MAS ensures the solution of complex scattering problems with predesigned accuracy and the detailed determination of both the near and far fields.

6. CONCLUSIONS

In this work, we have described the conventional method of auxiliary sources (MAS) in application to 2D and 3D scattering problems upon the bodies of complicated shape and filling. Next, we offered general recommendations for the proper implementation of the MAS with predesigned accuracy. Finally, we illustrated the application of the MAS to particular problems, including the problems of anisotropy, chirality and those with multiply connected boundaries. Far and near fields for different situations have been analyzed through numerical simulations in a wide frequency band starting from the quasi-static up to the quasi-optics. The

efficiency of the MAS to study complex scattering problems, as well as to visualize various physical phenomena in electromagnetic and light wave band has been demonstrated.

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