

Application of the Method of Auxiliary Sources to Toroidal Resonator Study

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Abstract— The Method of Auxiliary Sources (MAS) is used to study the torus form concave-convex hollow metallic voluminous resonator. Such resonators have wide application in high frequency radio-electronics. Using the proposed method it is possible to find all eigen frequencies of the voluminous resonators and study the structure of the eigen resonant fields.

Keywords— MAS, toroidal resonator, resonance, eigen frequency, eigen field.

I. INTRODUCTION

The aim of the presented study is the development of a MAS based methodology for the complex shape metallic cavity objects' resonant characteristics study. As an example round concave-convex toroidal resonator is considered.

Metallic cavity voluminous resonators have wide application in modern high frequency radio-electronics. They are used in many kinds of generators and oscillatory circuits, in microwave up to VHF frequency range. For example, in electronic generators klystrons, magnetrons, tube based, solid state and so on voluminous resonators with high quality factor, act like stabilizers of microwave frequency generators. Resonators with high quality factor can be used as the filters in high frequency electronics.

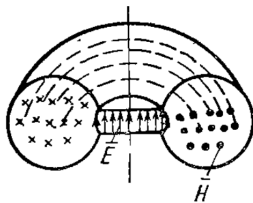


Figure 1. Toroidal resonator

Toroidal resonators with circular (or rectangular) form of vertical section (Figure 1.) have the biggest spreading. The main uniqueness of the oscillations inside them is sharply expressed special separation of the electric and magnetic fields. Resonant electric field is concentrated in the middle part between the planar part of the surface and the magnetic field is concentrated in the round part. This gives ability to use the method of the equivalent scheme for approximate calculation of the main frequency using the known formula

$$\omega_0 = 1/\sqrt{LC}.$$

At the same time because of the complex shape, the value of the inductivity L and the capacity C could be found using the approximate formulas. This increases the error of calculation. Besides this it is very complicated to find the next, higher eigen frequencies and, a specially, it is impossible to investigate the eigen resonant field distribution.

Nowadays the method of equivalent scheme is one of the main method for practical analysis and parameter determination for complex shape voluminous resonators [1-3]. The strict analytical calculations are possible for very simple cases [4-9]. As it is well known the problem of resonant frequencies and resonant field's determination for voluminous resonators is related to the mathematical problem about eigen values of the linear differential operator with homogenous boundary condition. If we look for nontrivial solution as a series by Green function of this operator, relating to its eigen values form the condition that corresponding determinant must be zero, we get some transcendent equation. The calculation difficulties arise during determination of these zeros [10].

In [11, 12] a very simple and effective method for eigen values problem solution based on MAS was described. The main idea was to solve the diffraction problem of the electromagnetic wave for the voluminous resonator and construct corresponding reradiated field's frequency characteristics. There was shown that some maximums arise and these maximums of the frequency characteristics correspond to the eigen resonant frequencies.

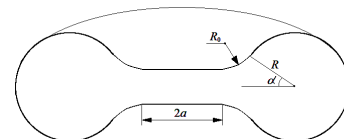


Figure 2. Resonator's cross section

In the given work this method is used in order to investigate resonant properties of the toroidal resonator. Some features of numerical simulations are discussed. The cross-section of this resonator has the form shown on the Figure 2. The difference with the Figure 1 is that the planar part connection to the torus is rounded. Geometry of the torus could be changed by its parameters: a , R_0 , R , α . The proposed approach based on MAS gives ability to valuably study the characteristics of the given resonator. After the description of the corresponding theoretical part, there is given the analysis of some obtained numerical results.

II. THEORETICAL PART

The diffraction problem of a harmonic electromagnetic wave on the object, which represents a cavity with a toroidal shape ideally conducting closed surface S is considered (Figure 3). The point source of the incident wave $\vec{E}_{inc}$ with the given polarization $\vec{p}$ is located in the point with radius vector

$\vec{r}_0$. During the numerical solution of the given problem using MAS, instead of the continuous surface S we consider discrete amount of the collocation points N . As a result, the incident field can penetrate inside the object and at the eigen frequencies the resonant field with high amplitude is formed interior. On the non-resonant frequencies the inner field is very small.

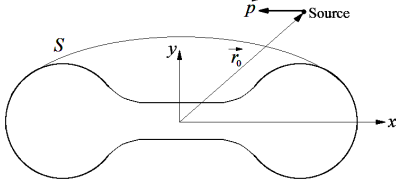


Figure 3. The diffraction problem

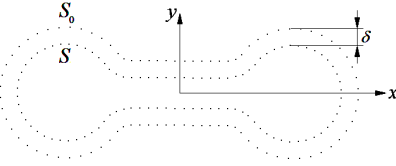


Figure 4. MAS application

If $\vec{\mathcal{R}}$ is unknown exact scattered field, its approximated value we find as a sum of the auxiliary sources' fields.

$$\vec{E}(\vec{r}) = \sum_{n=1}^N a_n \vec{E}_n(\vec{r}_n, \vec{r}). \quad (1)$$

The auxiliary sources are distributed on the outer auxiliary surface S_0 , which is shifted by the δ distance from S (Figure 4). In the expression (1), $\vec{r}$ is the radius-vector of outer source, $\vec{E}_n$ - is the field of the auxiliary source, and $\vec{r}_n$ - is the radius-vector of this source. The unknown amplitudes a_n of the auxiliary sources are determined from the boundary condition.

$$\vec{E}(\vec{r}_m^s) \cdot \vec{\tau}_m = -\vec{E}_{inc}(\vec{r}_m^s) \cdot \vec{\tau}_m, \quad m=1, 2, \dots, N, \quad (2)$$

where $\vec{r}_m^s$ is radius-vector of collocation point on the surface S , and $\vec{\tau}_m$ is the tangential vector. $\vec{E}_n$ aggregate is total linearly independent system of function in L^2 . In case of the correct selection of auxiliary parameters N and δ this makes the minimum of the norm

$$\left\| \vec{\mathcal{R}} - \sum_{n=1}^N a_n \vec{E}_n \right\| \approx 0. \quad (3)$$

If we change the frequency f of incident wave continuously at some frequencies the condition (3) is not satisfied. This can be explained by the fact that the field (1) at these frequencies contains not only diffracted field but also resonator's eigen field. The last one can penetrate through the space between collocation points. This increases the total reradiated power. The reradiated power can be calculated using the formula:

$$W(f) = \frac{1}{2} \int_{S_w} \text{Re}(\vec{E} \times \vec{H}^*) \cdot d\vec{S}_w, \quad (4)$$

where S_w - is closed surface, which contains the investigated area. The magnetic field $\vec{H}$ is determined from the Maxwell's equation:

$$\vec{H} = -\frac{i}{\omega\mu} \text{rot} \vec{E}. \quad (5)$$

We have to mention that the MAS has its specifics for this particular problem. If we increase the number of collocation points on the resonator's surface, this increases its quality factor and as a result the resonant graph becomes narrow (Figure 5).

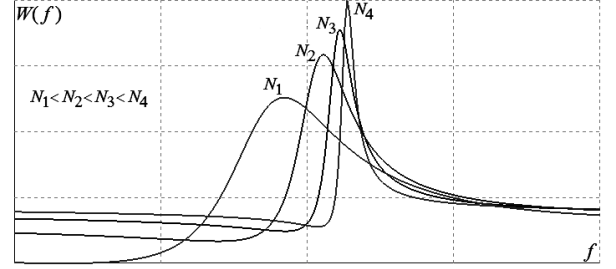


Figure 5. The dependence of the resonance graph form on the collocation point number

This makes possible to increase the number of collocation points step by step. At each step the range of resonance is decreased. This makes possible to find all eigen frequencies.

III. RESULTS OF NUMERICAL SIMULATIONS

When we use the MAS, it is important to correctly choose the auxiliary parameters N and δ , considering the location of the field's singularities. The accuracy of calculation, the stability of the obtained results, and its reliability depends on this correct selection. The given toroidal resonator has very complicated, concave convex form, as the result of this the inner resonant field has singularities, which are distributed outside area in the center of the rounded part. That's why the shift distance δ of the outer auxiliary surface was chosen less than radius of rounded part R_0 (Figure 2). While conducting numerical experiments using MAS the auxiliary parameters N and δ were selected so that the mean squared error of boundary condition satisfaction was less than 15-20%.

The main geometric parameters of the considered resonators are radius a of the planar part, radius R and the angle α of the torus open part and the radius R_0 . For them there was chosen the next initial values: $a = R = 0.6$ m, $R_0 = 0.7R$, $\alpha = \pi/5$. The values of the auxiliary parameters $\delta = 0.5R_0$, $N = 2000$ gave the above mentioned mean squared error.

During the investigation, the interest was focused on the eigen oscillations of high order, also how the geometric parameters of the resonator affect the frequency characteristics of reradiated field $W(f)$. The investigation showed that the small increase of the radiuses a and R negligibly shifts the maximums of frequency characteristics along the horizontal axes. If we change the angle of the torus's open part the values of the maximums are changed.

On the Figure 6 the frequency characteristics for three different values of the angle $\alpha = \pi/5.1$, $\alpha = \pi/5$ and $\alpha = \pi/4.9$ in the frequency range 0.37GHz – 0.38GHz are shown. As we see the biggest value of the maximums is at the angle $\alpha = \pi/4.9$.

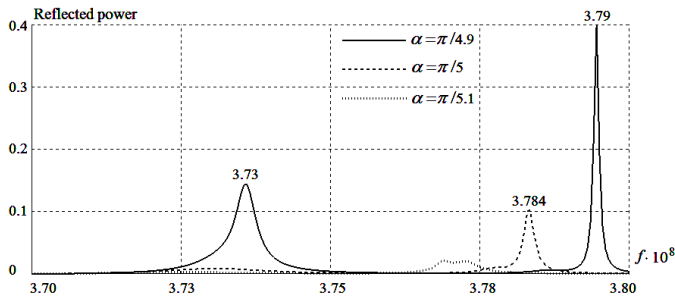


Figure 6. Frequency characteristic

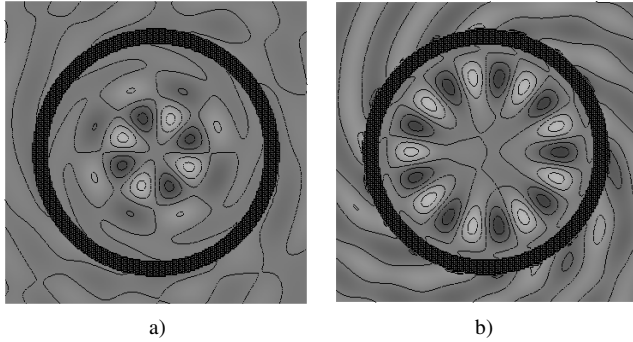


Figure 7. Resonant fields: a) $f=0.373\text{GHz}$, b) $f=0.379\text{GHz}$

On the Figure 7 there are given the near field distributions in the horizontal section, for two maximums on the Figure 6. As we see, the first maximum is the resonance of the objects' inner planar parts, where the four wave length is located. The second resonance at the frequency 0.379GHz corresponds to the outer toroidal part. Along the torus nine wave lengths are located. It is interesting to continue the investigation and choose the parameters so that combine these two resonances.

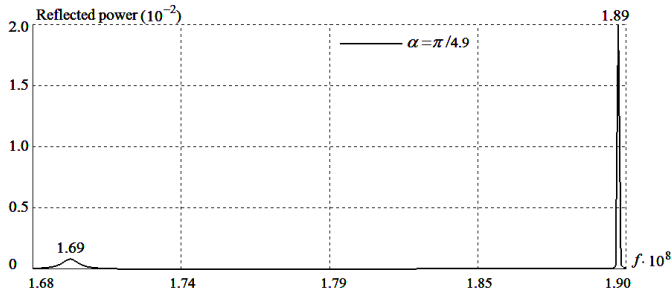


Figure 8. Frequency characteristic

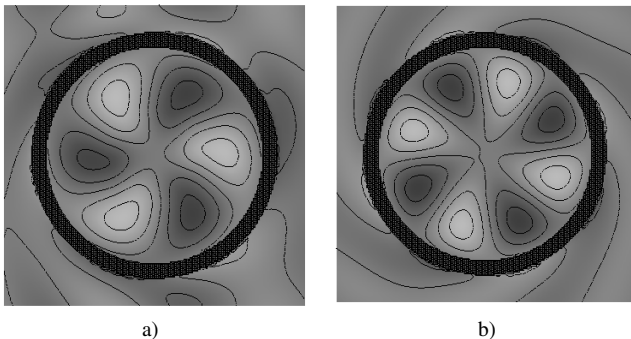


Figure 9. Resonant fields: a) $f=0.169\text{GHz}$, b) $f=0.189\text{GHz}$

The analogical investigation was realized for low frequencies in the frequency range $0.17\text{GHz} - 0.19\text{GHz}$. The corresponding frequency characteristics are given on the Figure 8. The resonant maximums are observed near the borders of this frequency range. The distribution of the resonant field is given on the Figure 9 a) and b). This case is different from the previous one by the fact that both resonances corresponds to the resonance of the outer toroidal part.

We have to mention that at the animation of the resonant field a) and b) on the Figure 6 and Figure 8 the resonant fields rotate. The fields' rotation on the Figure 6 and also the field b) on the Figure 8 was anti clockwise. The rotation on the Figure 8 a) was clockwise. We have tried to improve the geometric form accuracy during computer simulation but this effect is still observed. This gives the idea that may be this effect is realistic and the reason of this is the superposition multiple reflections with different phases. It is clear that we have to conduct the next investigations in this direction. The analogical investigation could be realized for dielectric resonators, also for metallic resonators with dielectric inside.

IV. CONCLUSION

The MAS is used for toroidal resonator resonant properties investigation, which has wide practical application. The investigations were provided in the frequency range of the high order eigen oscillations. We found some resonant frequencies and there was studied the resonant (eigen) fields at these frequencies. Some features of numerical simulations were discussed. In future we are going to study more complicate geometry shape figures.

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