Wave Field Singularity Aspects in Large-Size Scatterers and Inverse Problems

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***Abstract*—It is known that any scattered wave field carrying energy into infinity must have source singularity centers within a bounded space. Otherwise, the scattered field should be identically equal to zero everywhere [1]. In this paper, attention is paid to localization of these singularities under the assumption that every scattered wave is determined uniquely by its own singularities. Investigations have shown that these singularities are distributed as "bright centers" and the distance between them depends on frequency. To determine the position (localization) of the scattered wave field singularities, the functions describing converging and diverging waves are used. Based on these concepts and the method of auxiliary sources, an efficient numerical method to reconstruct a field up to its singularities is suggested. The localization of singularities is used for partial representation of the scattered fields, which reduces significantly the number of unknowns in describing the scattering process and leading into optimized inverse scattering problem solutions.**

***Index Terms*—Auxiliary absorbers, auxiliary sources, scattered field singularities.**

I. Introduction

THE investigation of diffraction and scattering phenomena concerning three-dimensional (3-D) objects is highly important in many practical problems and has received a considerable amount of attention in recent years. Development of efficient numerical methods is of high practical interest. In many technology applications, modern computer resources still restrict utilization of well-known methods such as method of moments, finite-element method, finite-difference time-domain (FDTD), and other techniques because of memory and CPU-time limitations. Reduction of the number of unknowns and review of previous work on this topic is given in [2]. Other approaches are based on a combination of various techniques, which are known as hybrid methods.

This paper presents the minimization of required computer resources using the method of auxiliary sources (MAS) in solving two-dimensional (2-D) problems. Similar considerations are applicable for 3-D problems as well. The propositions given below are also related to the solution of an inverse scattering problem, particularly with antenna pattern synthesis. Many papers on antenna synthesis have been published because of its wide applications in microwave communication systems, array synthesis, and adaptive antenna design [3]. A review of the relevant literature can be found in [2] and [3].

The MAS was developed to solve a wide class of electrodynamic problems and was improved in the effort to solve numerous practical problems [4]–[6]. The MAS was mathematically justified by Kupradze [7]. There are also a number of authors who, probably independently, proposed such representation of the scattered fields [8]–[13].

The traditional practice was to use the MAS for boundary problems of mathematical physics without optimization of its auxiliary parameters. It has been shown in [4] and [5] that basic difficulties arise unless all physical properties of scattered field singularities (SFSs) are taken into account in the algorithm. The stability and SFS are stated. In Section III, a new algorithm based also on the MAS, SFS localization, and field reconstruction based on the analytical continuation of the scattered wave fields is presented. To this end, in 2-D problems, the functions $H_0^{(1)}(kr)$ and $H_0^{(2)}(kr)$, describing outgoing and incoming waves, respectively, are used. It is shown that SFSs are distributed as "bright points" near the caustic surface. These positions of SFS can be used to achieve an efficient solution of scattering problems concerning large objects by partial representation of scattered wave field.

In Section IV, the application of the MAS and SFS aspects to treat inverse scattering problems is examined. This provides an interesting optimization technique in antenna pattern synthesis. In this context, the design of an optimum radiator is based on minimizing the reactive field in near zone leading into a well-matched antenna with a predefined radiation pattern.

II. MAS and SFS

Consider the 2-D scattering problem shown in Fig. 1, where the boundary surface S is a perfect electric conductor (PEC) and the area within the surface is denoted as $\mathbf{D}$. We must determine the electric and magnetic fields outside of $\mathbf{D}$ satisfying the wave equation and the boundary condition on the surface S under illumination of an incident wave U^i. Assuming an $e^{-i\omega t}$ time dependence of the field quantities, the scattering problem is reduced to finding the solution of Helmholtz equation

$$\Delta U^S(x,y,z) + k^2 U^S(x,y,z) = 0 \tag{1}$$

and the implementation of the boundary condition

$$W\left\{U^s(x,y,z) + U^i(x,y,z)\right\} = 0, \quad M(x,y,z) \in S \tag{2}$$

where $U^s(x,y,z)$ is the scattered field (SF), $U^i(x,y,z)$ is the incident field, and W is the operator of the boundary conditions,

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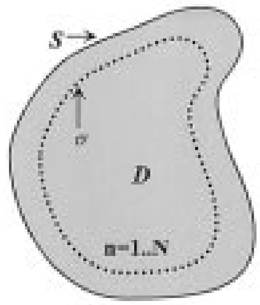


Fig. 1. MAS geometry for 2-D perfectly conducting objects. The auxiliary sources are placed uniformly along the auxiliary surface σ inside the nonphysical area of the scatterer D. Surface S is the perfectly conducting bound of area D.

which in the case of PEC calculates the tangential to the surface S component of electric field. Then, according to [7], an auxiliary surface σ is introduced inside area $\mathbf{D}$, and a set of sources positioned at points $\{x_n, y_n, z_n\}_{n=1}^{\infty} \in \sigma$ are uniformly distributed as shown in Fig. 1.

Let $\{U(|\vec{r}_n - \vec{r}|)\}_{n=1}^{\infty}$ be the Helmholtz equation solution associated with elementary sources by which the scattered field is to be represented. The relevant functions are

$$U(|\vec{r}_n - \vec{r}|) = H_0^{(1)}(k|\vec{r}_n - \vec{r}|) \quad \text{for 2-D case} \tag{3}$$

$$U(|\vec{r}_n - \vec{r}|) = \frac{e^{ik|\vec{r}_n - \vec{r}|}}{|\vec{r}_n - \vec{r}|^2}(\vec{r}_n - \vec{r}) \quad \text{for 3-D case.} \tag{4}$$

The following have been proved [7].

1) The set of functions $\{\{U(|\vec{r}_n - \vec{r}|)\}_{n=1}^{\infty}\}$—elementary auxiliary sources (ASs) that describe the characteristics of the field (electric field, magnetic field, or field potentials)—is complete and linearly independent on the surface S in L_2 space.
2) There are coefficients j_n such that, using the first N functions of the aforementioned system, all kinds of functions of SF on the surface $\mathbf{S}$ can be represented by a linear combination of the fundamental solutions with the appropriate coefficients. For a perfectly conducting scatterer

$$W\left\{\sum_{n=1}^{N} j_n U(|\vec{r}_n - \vec{r}|)\right\}\Bigg|_S = -W\left\{U^i(x,y,z)\right\}\big|_S. \tag{5}$$

Then, the approximate solution of the boundary problem outside $\mathbf{D}$ is

$$\tilde{U}^s(x,y,z) = \sum_{n=1}^{N} j_n U(|\vec{r}_n - \vec{r}|) \tag{6}$$

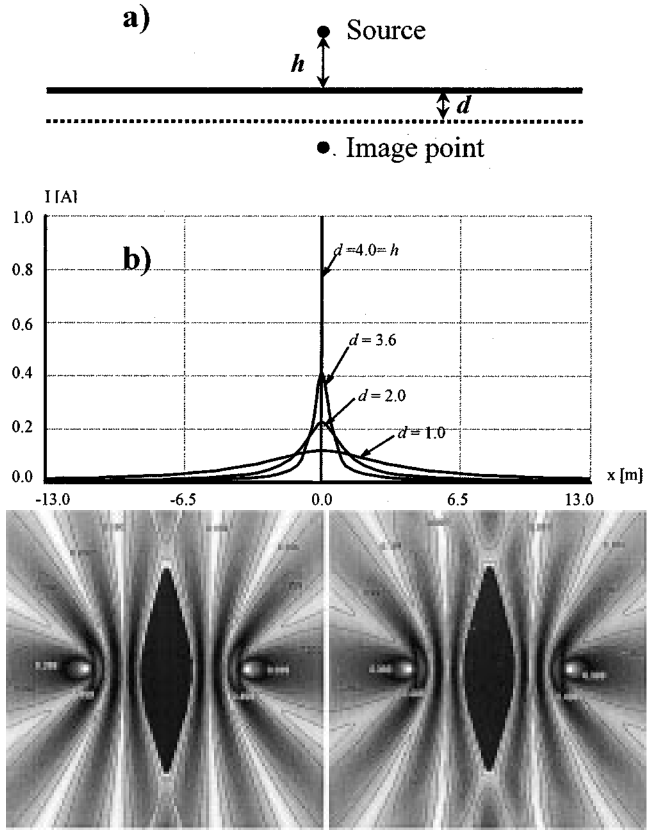
which will approach exact solution $U^s(x,y,z)$ as $N \to \infty$.

This is the essence of the conventional MAS introduced by Kupradze [7].

Previous investigations [4]–[6] have shown that the correct choice of the auxiliary contour and the placement of the AS are important factors to achieve efficiency. Particularly, if SFS location is not considered, the computing process might diverge. The necessary number of terms to be taken into account in (5) depends strongly on the form of the auxiliary contour and location of AS. When the auxiliary contour moves away from the physical surface S, the required number of terms in (5) to achieve a specific accuracy decreases sharply, and consequently the required CPU-time also decreases [4]–[6]. This can be explained by the fact that shifting of AS inside the scatterer makes the SFs function more smooth on the surface of the body. The compensation of incident field at the collocation points with the scattered field is elaborated, i.e., the fulfillment of the boundary conditions between collocation points is improved. However, care must be taken since further deepening of AS could lead to a divergent solution. This happens if the SFS appears outside of the auxiliary contour and the fields generated by AS cannot describe these singularities, because the SF is analytical everywhere outside the auxiliary contour. A highly vivid example is the problem of an infinite planar conducting surface illuminated by a point source placed at some distance, i.e., $h = 4$ [Fig. 2(a)]. The auxiliary current source distributions for various distance d of auxiliary contour from the planar surface are shown in Fig. 2(b). It is observed that at a particular distance $d = h$, the sharpest amplitude of current at the center is obtained. Indeed, this corresponds to the well-known mirror image of the exciting source. It means that the exact solution is obtained when the auxiliary contour passes through a certain point—the point where the mirror image of the source is located. This is the point where the SFS is located and only one AS is sufficient to represent the SF everywhere above the plate. If the auxiliary contour is located deeper beyond the SFS point, the solution diverges.

The following cases were studied to investigate the dependence of this type of singularity point, to trace how this single mirror image point is transformed into caustic surface in the case of large complex structures, and to verify the capability of using the minimum number of sources for efficient representation of scattered fields.

1) Hyperbolic surface illuminated by the point source placed in its virtual focus. In this case, the main singularities are located in the real focus of hyperbola. It was shown in [14] that by only one AS placed in this focus, the scattered field can be determined with good accuracy. The accuracy depends on the wavelengths and parameters of the hyperbola. It is improved when frequency increases and when the hyperbola is transformed to the plane surface. With the help of this approach, the scattering at the cylinder formed by two symmetrically placed hyperbolic surfaces with common focus was solved when they were illuminated by two symmetrical sources placed in the outside foci of the hyperbolas. For $ka = 20$, where a is the half linear size of the scatterer and k is the wave number, the conventional approach of the MAS needs about 400 ASs (unknowns), whereas consideration of SFS gives the same accuracy using only one AS placed in the common focus of the hyperbolas. Accuracy in the latter case is improved with increasing frequency. The main difference from exact solution occurs near the edges of the scatterer that are not taken into account by this single AS. It does not exceed 1% for $ka = 20$ and above. The results of the described solution are presented in Fig. 2(c). The scatterer is shown in black and lines of equal amplitudes of the electric field are presented in both cases for compression.



(c)

Fig. 2. (a) The line source, placed at distance h, is illuminating a perfectly conducting plane (solid line), while the auxiliary source loci are taken to be a line at distance d (dotted line). (b) Calculated amplitudes of auxiliary sources for various distances d of auxiliary surface. (c) Diffraction by two hyperbolic surfaces, E-field distribution; (left) solution is found by means of a single source placed in the inner focus of the hyperbolas, (right) solution is found by exact calculation. In both cases, incident field is formed by the sources placed in the outer foci of the hyperbolas. (Half linear size of scatterer $a = 6.35\lambda$, where λ is wavelength.)

2) Conductive cylinder illuminated by two symmetrically placed elementary sources [5], [15].
3) Two half-infinite spaces of different dielectric constants excited by a linear source located near the interface [15], [16].
4) The geometry of case 3) including a dielectric object in one of the two half-infinite spaces [15], [16].
5) A large radius conductive cylinder excited by a plane wave [15].

In each specific case, the dependence of SFS on the incident wave frequency has been investigated. Some of the cases are discussed below.

An important issue is the critical dependence of SFS in treated geometries and especially the movement of SFS loci with the transformation of related surfaces such as the modification of a planar surface into a cylinder and replacement of a conductor body with a dielectric.

All these examples have shown that the SFSs are distributed along the caustic surface as "bright points." In the following, it is shown that the placement of the auxiliary sources in these points sharply reduces the number of unknowns under a required accuracy and is similar to the mirror images method for large values of ka.

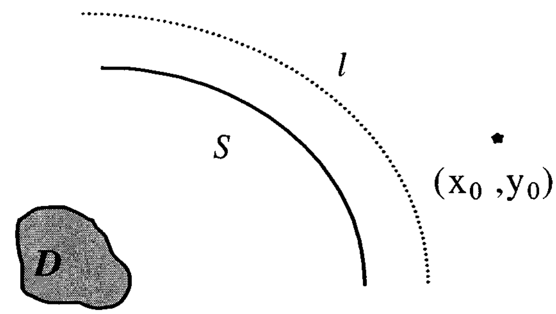


Fig. 3. Scattered field reconstruction method. The auxiliary absorbers placed along surface l are used to reconstruct the scattered field measured along surface S. The scatterer D is illuminated by a line source placed at origin (x_0, y_0).

III. Application of MAS in Field Reconstruction and SFS Visualization

It is known that the outgoing field outside the scatterer is regular and is uniquely determined by the associated source singularities located within the scatterer or on its edges (edge singularities) [1]. Consequently, the field values in this regular scattered field region and additionally up to the singularity region nearest to the physical surface could be determined using suitable, analytical in the corresponding area, wave functions.

There are many methods of reconstruction of the analytical continuation of scattered wave field. Mathematical justification of those approaches is given in [18] and [19]. There are other known approaches in the direction of wave field visualization presented in [20]. In the present holographic method, the wave field reconstruction by known data of SF amplitudes and phases on some surface in a neighboring area is implemented. The suggested method is based on analytical continuation of the wave field computed also by MAS. A new idea to use the functions describing the field propagating toward the source is introduced here. Such a function is $H_0^{(2)}(kr)$ in 2-D for the same time dependence of $e^{-i\omega t}$.

Assume a body (see Fig. 3) illuminated by a line source placed at (x_0, y_0). Then, assume that on a certain surface S at some distance from the object, the complex SF $U^s(\vec{r})$ is known (amplitude and phase). The aim is to reconstruct the analytical continuation of the SF around the object until one reaches the singularity points (sources of SFs or reemitters). To this end, as shown in Fig. 3, on the surface l, near the surface where the scattered field is known, ASs of the same frequency and arbitrary coefficients a_n are placed. The nature of these ASs is a matter of high importance. Since the ASs will be used to describe a field propagating toward them, the propagation vector of the SF should also be oriented toward the sources. Thus, these ASs should not obey radiation conditions; to the contrary, they should behave as absorbers, generating any arbitrary field $U(\vec{r})$ that propagates toward them as

$$U(\vec{r}) = \sum_{n=1}^{N} a_n H_0^{(2)}(kr_n). \quad (7)$$

Therefore the ASs are named auxiliary absorbers (AAs) under the assumption of the time dependence of $e^{-i\omega t}$. Then the known SF is matched with the field of AA at the $M \geq N$ points on the surface S.

The corresponding linear system of equations is written as follows:

$$\sum_{n=1}^{N} a_n H_0^{(2)}(kr_{nm}) = U^s(r_m). \tag{8}$$

The solution of the system (8) determines the amplitudes and phases of AAs.

Because of the uniqueness of analytical continuation of the scattered wave field, the AAs will restore the field up to the main singularity points within the nonaccessible area of the conducting object. In the case of semitransparent objects, this method will restore the main SFS inside the body. In the case of a diffuse reflector or Lambert surface [21], this method will restore the surface of the object since SFSs are located on the surface of the object.

To verify this concept, an experiment was carried out to reconstruct the SF from the conducting cylinder of 2 cm radius [22], which was illuminated at a frequency of 30 GHz by a horn antenna at approximately a distance of 30 cm. The SF was recorded in far zone at distance of 30 cm from the cylinder by a dipole sensor of 5 mm connected to the HP8510C vector network analyzer. Complex values of the scattered field were measured on a half-circle in the backward direction. The measured data were processed to reconstruct the analytical continuation of the wave field by the method described above. The AA were placed on a half-circle of 30.5 cm radius next to the one used for measurements as shown in Fig. 4(a). In this figure, a minimum leveling of the presented reconstructed field is applied to localize all possible SFSs. In Fig. 4(b), a zoom in Fig. 4(a) without leveling is also shown. In both figures, the dashed line presents the position in the 2-D space of the cylinder. It is evident that the method enables one to localize the auxiliary sources inside the scatterer and thus provides an efficient and highly simple target imaging technique.

The same reconstruction algorithm is also valid for imaging purposes using ultrasound waves. Experimental measurements were carried out using ultrasound waves of 50 kHz. A dielectric cylinder (plastic container) of 3.5 cm radius filled with oil was placed in a water tank. Then the cylinder was illuminated by an ultrasound source at 50 kHz frequency. The scattered field was measured in the points on the black arc shown in Fig. 4(c). The scatterer was shifted out of the center of the arc, whose aperture is 90°. Then applying the algorithm described above, the measured data were used in reconstructing the field. In Fig. 4(c), the amplitude distribution of the reconstructed field is shown. The darker the area, the higher the amplitude of the reconstructed field. The maximum value corresponds exactly inside the nonphysical area of the scatterer where the SFS should be. The experiment described above demonstrates the applicability of the proposed reconstruction in the case of penetrating objects.

The uniqueness property of the described field reconstruction method states that the computed visualization pattern does not change if additional sources are placed outside the region between the scatterer and the visualization area. This is true as in any holographic methods where neighboring sources do not disturb the obtained picture.

Therefore, the application of the described algorithm exhibited the capability of field reconstruction when the field is known on a specific curve. Attention is paid to the SFS localization for a perfect conductor circular cylinder of large radius compared to wavelength when a plane wave is incident.

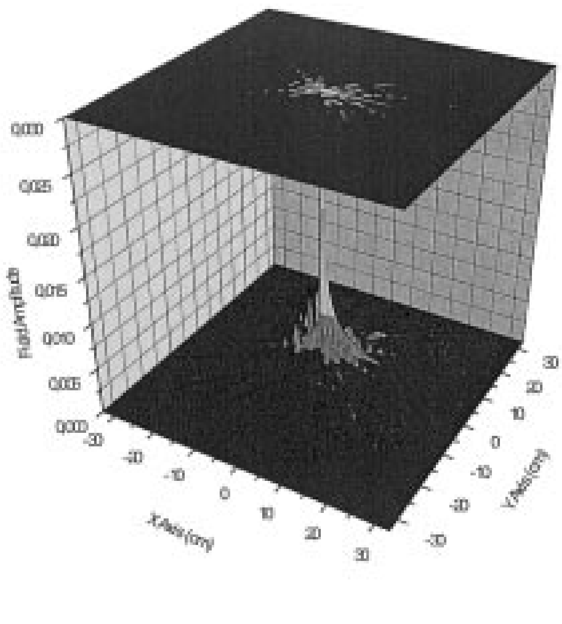
Following a thorough numerical study, the results shown in Fig. 5 are obtained. The observed results are summarized as follows.

1) It is evident that as shown in Fig. 5, the SFSs are distributed in discrete points, like "bright illuminators" located at same distance from each depending on frequency. We call these points "bright" because using the procedure described above, they are found in place where the restored field has a sharp increase of amplitude.
2) The SFSs are distributed close to the caustic surface (SFS of geometric optics), on the average approaching the caustic curve $x = ((1/2) + (y^2/R^2))\sqrt{R^2 - y^2}$, $y = y_0^3/R^2$ (where R is the cylinder radius, y_0 is the distance of the incident ray from the x-axis) of the cylinder when the frequency tends to infinity. Determination of a caustic surface with a detailed explanation can be found in [19].

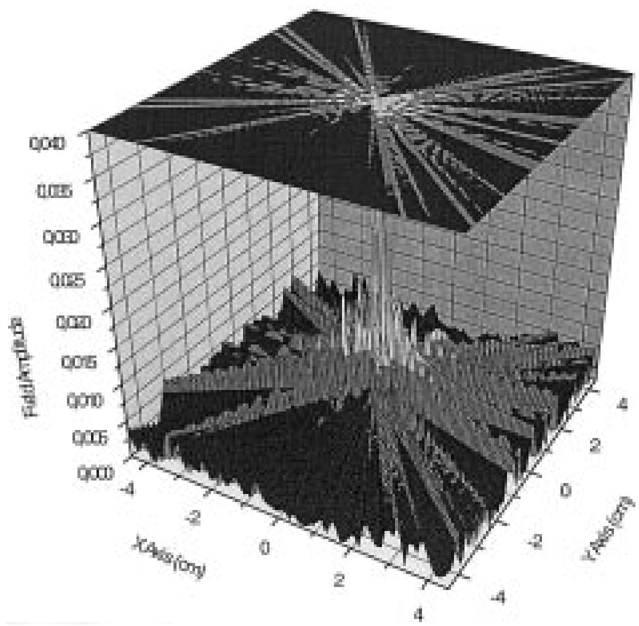
In fact, the above-mentioned SFS distribution of the circular cylinder for the fixed wavelength can also be obtained with high accuracy based on following consideration. The scatterer surface beginning from the nearest point is divided into parts on which the phase change of incident field is equal to π, i.e., the entire surface is divided into Fresnel zones [19]. For each of these zones, the middle point on the surface is chosen and the corresponding point on the caustic surface is determined. The set of caustic points obtained by this procedure is similar to that shown in Fig. 5. As a matter of fact, no surface of a perfectly conducting cylinder is observed, and the described points are those that one can actually see. The same case is shown in Fig. 2. One can see just the mirror image of the source.

Notice that the picture of SFSs is obtained using two different approaches. It is concluded that the number of given "bright spots" or number of equations equals the whole number of $N \approx 2ka/\pi + 1$. So the localization of SFSs can be determined based on scatterers' geometry and the nature of the incident field is more accurate, i.e., dividing the whole scatterer into the Fresnel zones and finding the corresponding points on the caustic. It is important that this can be done before the solution of the scattering problem.

Therefore, placing at the obtained points the ASs that satisfy the boundary condition on the scatterer surface, we will reconstruct the SF if the assumption is made that each source radiates in the sector of the corresponding zone. This means that the scattered field is represented partially by each corresponding source. Of course, the exact boundary condition is fulfilled in the middle point of each zone (where this was forced), but the source at the caustic point satisfies on average the boundary condition in the best way. This originates from the definition of the caustic point itself. Moreover, the error of



(a)



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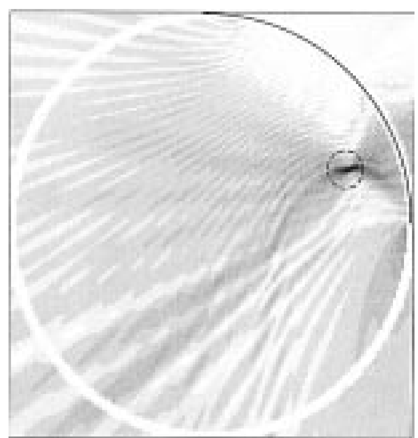
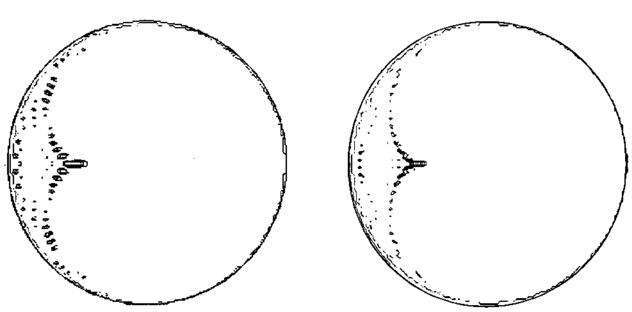


(c)

Fig. 4. (a) The reconstruction of the scattered field using scattered field measurements on a half-circle at distance 30 cm of the 2 cm radius cylinder placed at the origin (0,0). The field is incident along the x-axis with the radiating antenna placed at (30,0). (b) Magnification of Fig. 4(a). The reconstruction of the scattered field using scattered field measurements on a half-circle at distance 30 cm of the 2 cm radius cylinder placed at the origin (0,0). The maximums of the reconstructed scattered field are all placed inside the nonphysical area of the cylinder, denoting the scattered field singularities. (c) Ultrasonic field reconstruction experiment. The amplitude distribution of the reconstructed field. Black arc corresponds to the points where the scattered field was measured during the experiment. The dark region shows where the illuminated sample was located.



a) $ka = 20\pi$ b) $ka = 40\pi$

Fig. 5. The scattered field singularities for a perfectly conducted cylinder illuminated by plane wave propagating along x-axis. Two cases are considered: (a) $ka = 20\pi$ and (b) $ka = 40\pi$, where k is the wave number and a is the radius of the cylinder. The SFSs are distributed along the caustic surface.

fulfilling the boundary condition in the border of contiguous zones is the same from two corresponding sources. Thus the whole field of all sources is continuous despite their sector radiation. One should also note that in the mentioned case, the phase difference of the incident field between any two neighboring points where the boundary conditions are satisfied is 180° and between the corresponding sources on caustic surface $-90°$, i.e., α, $\alpha + 90°$, $\alpha + 180°$, etc., so they are mutual orthogonal. Therefore, the sources form the wave, propagating along the caustic surface.

In Fig. 6, the SF calculated by the algorithm described above for an elliptical scatterer illuminated by the plane wave $(ka = 100)$ is shown when the angle of incident field is $-30°$ to the big semiaxis. The semimajor and semiminor axes of the ellipse are $a = 1$ and $b = 0.2$. The number of ASs, as well as the number of

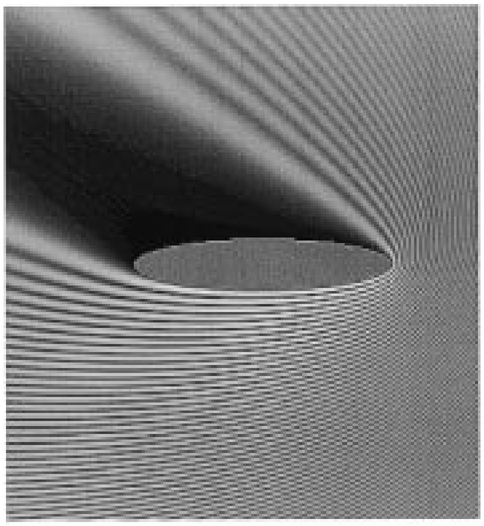


Fig. 6. The absolute value of the total field obtained by the partial representation of the scattered field using the auxiliary source placed in the phase centers of scattered field (SFS). $(ka = 100, N = 60)$.

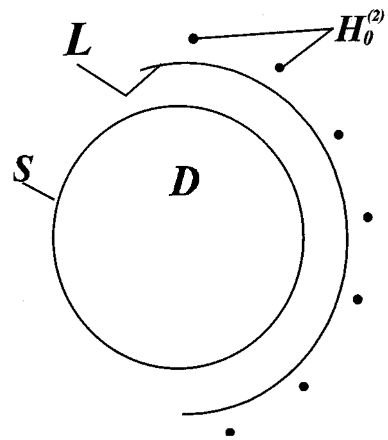


Fig. 7. The geometry of the near-field reconstructing algorithm. S is the surface where the auxiliary sources are distributed. The amplitudes of the auxiliary sources are calculated by matching the asymptotic expression of the auxiliary sources with the given far-field pattern. The near field outside D can then be calculated. L is the surface where the field of the absorbing sources is matched with the field of the auxiliary absorbers placed near surface L (dots).

employed equations, is 60. This provides the necessary accuracy of satisfying the boundary condition within 1–3%. The propagation constant easily could be increased, but it was limited with $ka = 100$ to avoid the problem of image resolution. Further division of each Fresnel zone into two or more parts can also increase the precision of the solution. As concerns the shadow zone, the field in it can be calculated by the set of ASs placed in the scatterer and tying up the incident field at the backside.

Thus the knowledge of the position of SFSs allows significant reduction of numerical cost and provides the possibility of computing structures up to the optical waves region $(ka > 200)$. The stability of the described procedure should be investigated in the solution of particular problems.

This method is applicable also to concave scatterers. If the scatterer is concave, the possible secondary reflections should be taken into account. The same is true when there is a system of two and more scatterers that complicate SFS distribution. In the case of dielectric bodies along with the caustic for reflected rays, the caustic for refracted rays should be taken into account to reconstruct the field within the scatterer.

IV. Inverse Scattering Problem Treatment

The method of field reconstruction described above can be successfully used to treat inverse scattering problems. There are many publications concerning various types and methods of solving inverse electrodynamics and antenna synthesis problems [23]. The general formulation of an antenna synthesis problem by its pattern has various approaches. There are also many approaches for optimization of the antenna shape and placement of its elements [23].

It is well known that the inverse problem does not have a unique solution. For example, for a specific pattern in an antenna design problem, different current distributions on the different surfaces can be introduced. The main problem discussed in this section is the design of an antenna occupying the minimum volume or surface that has a given far-field pattern and produces the minimum of the reactive field in near zone, so that the whole feeding energy is transmitted into a traveling wave. Such an antenna is called the "well-matched antenna."

It is assumed that in a scattering process, the energy being radiated into infinity should have its characteristic unique singularities in a limited region. On the other hand, it is known that a traveling wave is analytical everywhere except the region where the radiation sources exist. In the region outside the singularities, the radiated field develops the traveling waves and therefore the scattering patterns. Therefore, the problem is to find the location of these field singularities that generate the outgoing given waves. This is equivalent to localization of singularities of the specific far-field pattern and needs determination of the corresponding near field, as shown in Section III. To this end, an auxiliary circular antenna is introduced that can produce all kinds of patterns, including the given one (see Fig. 7). The far field of this antenna is matched with the desired pattern by distributing ASs on the auxiliary surface S inside the auxiliary antenna. It is obvious that the near field of this AS can be easily calculated using the method of field reconstruction, described previously. Therefore the singularities of the given pattern (i.e., the location of the antenna dipoles) are determined.

The case of 2-D is considered in detail. Consider the antenna design problem with the aim of providing a predefined pattern $F(\varphi)$, shown in Fig. 8. This pattern was originally generated by two electromagnetic-wave linear sources, placed at a distance of two wavelengths from each other. The near field of these sources shown in Fig. 9 is treated as unknown in this problem and is given only for comparision purposes. In the first step, we will obtain the mentioned near field corresponding to the specified pattern $F(\varphi)$ outside some area, which should surround all SFSs of the investigated wave field. This could be done by distributing N sources of $H_0^{(1)}(kr)$ type on some circular curve S surrounding the area D where the antenna is to be placed (see Fig. 9). In this case, S is chosen to be the circle of diameter d. It should be noted that diameter d must not be less than some definite value to provide the necessary width of the main lobe.

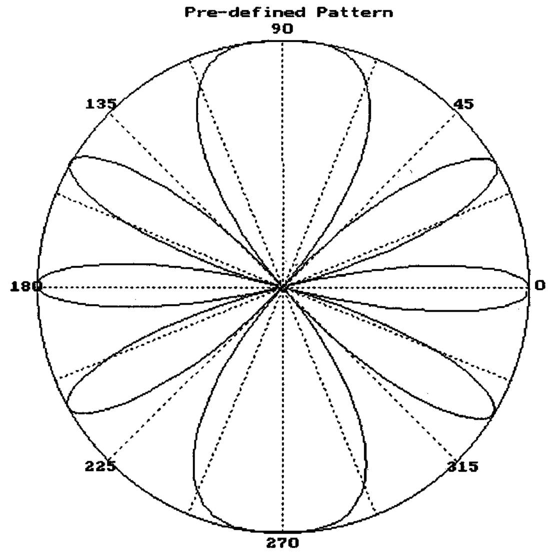


Fig. 8. Preset radiation pattern of unknown antenna sources (normalized pattern versus azimuth angle).

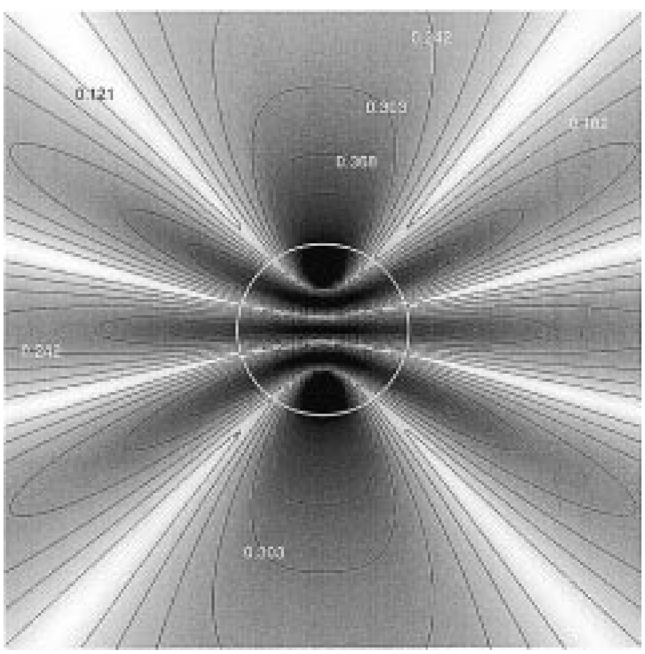


Fig. 9. Near-field distribution of the antenna pattern illustrated in Fig. 8. Circular line points to the region where all wave fields singularities are located and outside of which the near field is reconstructed using MAS.

This means that $d > \lambda/\Theta$, where λ is the wavelength and Θ is the width of the main lobe. The field radiated by these sources will be

$$E(r) = \sum_{n=0}^{N} a_n H_0^{(1)}(k|\vec{r} - \vec{r}_n|). \tag{9}$$

The far-field pattern will be determined by the asymptotic expression of (9), using the fact that

$$\lim_{r\to\infty} H_0^{(1)}(kr) = \sqrt{2/\pi kr}e^{-ikr+i\pi/4}.$$

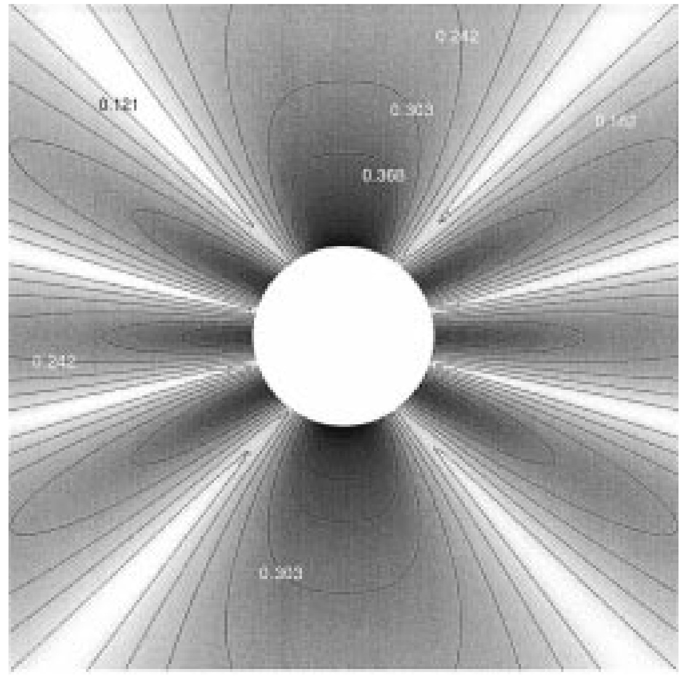


Fig. 10. The field reconstructed outside the circular region using the pattern data. All wave field singularities as well as auxiliary sources are enclosed by the circular region.

Therefore (9) becomes

$$\lim_{r\to\infty} E(r) = \sqrt{\frac{2}{\pi kr}} \sum_{n=0}^{N} a_n e^{-ik(x_n\cos(\varphi)+y_n\sin(\varphi))+i(\pi/4)}. \tag{10}$$

Applying the collocation method and satisfying the radiated field of these sources in M directions with the given field in the far zone, the system of linear equations will be obtained

$$\sum_{n=0}^{N} a_n e^{-ik(x_n\cos\varphi_m + y_n\sin\varphi_m)} = F(\varphi_m), \quad m = 1, 2, \ldots, M. \tag{11}$$

The number M should be enough to describe the desired pattern by (11), and in this case is equal to 360. Solving the system of linear (11), the coefficients a_n can be determined. These a_n are the complex amplitudes of the sources that generate the desired radiation pattern $F(\varphi)$. The precision of the solution depends on the number M of collocation points. This procedure was tested for different diameter d to ensure the stability and accuracy of the near field and pattern syntheses by currents distributed on the surface S.

It should be noted also that when the diameter d is such that the original sources appear in the points located on S, the solution of (11) gives sharp picks for currents in these points. That means that when some auxiliary sources are located in SFS points, they take the major contribution in forming the near field and pattern, as in the case described in Fig. 2. After generating the radiation pattern with the desirable accuracy, the fields of the sources are known everywhere outside the area D, including the near-field region (9). This field is shown in Fig. 10, which can be compared with that given in Fig. 9. It must be noted that

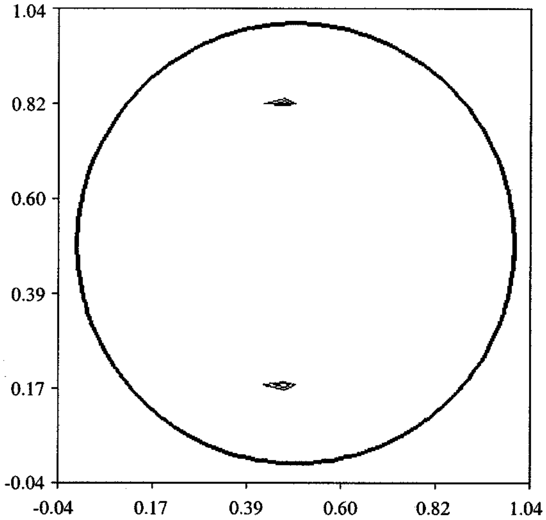


Fig. 11. Magnification of the circular region where the "bright points" of the reconstructed field are associated with the wave field singularities. This picture was obtained by the wave field reconstruction technique and provides information about the location of the sources, which radiate the predefined radiation pattern (Fig. 8).

the reactive field of these sources decreases as the diameter d increases. Area D should be large enough to provide the low reactive part of the field in near zone.

As the near field is known outside the area D, the next step is to continue it analytically inside D using the scheme of field reconstruction described above (Figs. 3 and 7). The extension of the near field also will be unique up to the singularities regarding the chosen area D. Let us choose some open curve L outside D, where the near field is known. Assume that N sources $H_0^{(2)}(kr)$ are placed at some distance from the curve L. These sources act as absorbers of the wave, traveling from the area D to infinity. The N chosen sources will reconstruct the field on the curve L if

$$\sum_{n=1}^{N} b_n H_0^{(2)}(k|\vec{r}_n - \vec{r}_m|) = E(r_m) \tag{12}$$

where b_n are the complex amplitude of n sources and $E(r_m)$ is the near-field value in the corresponding point on the curve L. If the number N is big enough, the reconstructed field tends to the real one. Since there is a matching of fields on the curve L, there also must be matching inside the area D up to the area of singularities of the given near field. So the numerical method of analytical continuation of the near field will be found inside area D. The result of the described algorithm is presented in Fig. 11. This picture shows the maximum amplitudes of the field reconstructed inside the D region, and these maximums are located in the points where the original sources were placed. Therefore, in the far-field pattern, there is the necessary information about field singularities, which are actually in the points of original sources, and one can note that this information was given only by consideration of the radiation pattern without previous knowledge of original sources. It is now obvious that by placing two sources in this area, the desired pattern can be obtained in most optimum way.

V. Conclusion

In this paper, recommendations are given to expand the possibilities of the method of auxiliary sources in solving large body scattering problems. The new direction is made by introducing the scattered field singularities as the areas where the auxiliary sources should be placed in order to minimize the computational cost. Specific examples are considered, and the improvements of the method when the SFS positions are known in advance are presented. The SFS visualization method is also suggested. For this purpose, "absorbers" (e.g., functions describing fields sinking to them) are used. The latter method is used for localization of the SFS for the cylinder, and the behavior of these singularities for different wavelengths is also examined. The described method of field reconstruction can be used to treat inverse electrodynamics and antenna synthesis problems as well.

The developed concepts of further exploration of the MAS significantly reduce computer resources in solving the scattering problems for large objects. The importance of the concept of SFS spatial localization is shown, and its application to inverse problems gives promising results and a new approach to their solution.

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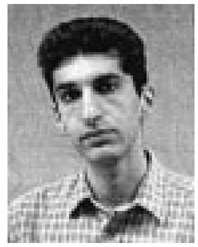

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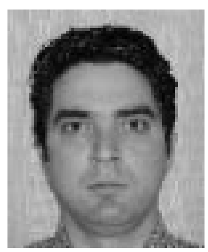

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