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PHYSICS

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Energy Determination of Scattered Field in 3D Diffraction Problems

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ABSTRACT. **Numerical experiments play an important role in modern practical problems of applied electrodynamics. On the one hand optimization of numerical methods for solution of the problem in real time is actual and on the other hand, it is necessary to estimate system's energy characteristics and study their behavior, which allows to check the accuracy of solution in terms of balance. An effective method for calculation of energy characteristics in 3D diffraction problem is presented.**

Key words: Hertz dipole, power.

Modern practical tasks have a complex character and require huge computational resources. Problems of applied electrodynamics are often reduced to the problem of finding the current distribution in space. There are several methods like M.O.M, M.A.S., F.D.M. etc. to solve this problem. Number of unknowns increases significantly along with the increase of the complexity of structures to be studied, thus requiring larger computational resources. In this article we present a method for determining energy characteristics of the system using MAS and MoM when solving diffraction problems.

Let us consider diffraction problem on the body excited by the Hertz dipole. Using the M.A.S, solution of the problem reduces to the solution of a system of linear algebraic equations, where unknowns are coefficients of the fundamental solutions of Helmholtz equation. Elementary dipoles (Hertz dipoles) are chosen as fundamental solutions in 3D electromagnetic diffraction problems.

It is well known that for a given wave zone Pointing's vector has the following form:

$$\vec{S} = \frac{1}{2}\mathrm{Re}[\vec{E}\vec{H}^*] \tag{1}$$

where $\vec{E}, \vec{H}^*$ are asymptotic values of corresponding fields (* denotes complex conjugation. Only the real part of averaged over the period has physical meaning).

To define energy characteristics of the system once the diffraction problem is solved, values of electric and magnetic fields in (1) are replaced by superposition of EM fields radiated by the elementary dipoles. Direct solution of this problem requires large amount of computational resources even for modern PCs. In this paper we have presented such analytical transformation of (1), which reduces computation time and increases solution accuracy significantly.

Asymptotic values of the fields radiated by the system of Hertz dipoles are [1,2]:

$$\mathbf{\vec{E}} \cong -\frac{k^2}{4\pi\varepsilon}\sum_{i=0}^{N}\left[\vec{R}_0\left[\vec{R}_0\vec{p}_i\right]\right]\frac{e^{ikr}}{r}, \quad \mathbf{\vec{H}} \cong \frac{k^2}{4\pi\sqrt{\varepsilon\mu}}\sum_{i=0}^{N}\left[\vec{R}_0\vec{p}_i\right]\frac{e^{ikr}}{r}. \tag{2}$$

Here $\vec{p}_i$ is a dipole moment of the *i*-th dipole, and $\vec{R}_0$ is a unit vector directed from dipole to the observation point. *N* is the number of dipoles (current elements). Zero term in sum (2) is the dipole moment of source. *k* is wave number of media. $\varepsilon = \varepsilon_0\varepsilon_r$ and $\mu = \mu_0\mu_r$ are permittivity and permeability of media respectively. After substituting (2) in

(1) and integrating (1) in the remote field zone we get the radiation power

$$P = \frac{1}{2}\int_s \mathrm{Re}[\vec{E}\vec{H}^*] = -\frac{k^4}{32\pi^2\varepsilon\sqrt{\varepsilon\mu}} \times$$

$$\times \mathrm{Re}\left(\sum_{i=0}^{n}\sum_{j=0}^{n} e^{i(\varphi_i-\varphi_j)} \int_0^{2\pi}\int_0^{\pi} \left(\left[\vec{R}_0\left[\vec{R}_0\vec{p}_i\right]\right]\left[\vec{R}_0\vec{p}_i\right]\vec{R}_0\right) e^{ik(R_0(\vec{r}_i-\vec{r}_j))} \sin\theta d\vartheta d\varphi\right). \tag{3}$$

After decomposition of vector product in (3) we get:

$$P = \frac{k^4}{32\pi^2\varepsilon\sqrt{\varepsilon\mu}} \mathrm{Re}\left(\sum_{i=0}^{n}\sum_{j=0}^{n} e^{i(\varphi_i-\varphi_j)} \int_0^{2\pi}\int_0^{\pi} ((\vec{R}_0\vec{p}_i)(\vec{R}_0\vec{p}_j)-(\vec{p}_i\vec{p}_j)) e^{ik(R_0(\vec{r}_i-\vec{r}_j))} \sin\theta d\vartheta d\varphi\right). \tag{4}$$

So, the problem is reduced to analytical calculation of the integral in (4). As a result of analytical transformations of (4) we get

$$P = -\frac{k^4}{32\pi^2\varepsilon\sqrt{\varepsilon\mu}} \mathrm{Re}\left[\sum_{i=0}^{N}\sum_{j=0}^{N} f_{i,j} e^{i(\varphi_i-\varphi_j)}\right], \tag{5}$$

where
$$f_{i,j} = \pi\left(f_{i,j}^1 \cdot \left((\vec{p}_i\vec{p}_j)-3p_{ij}\right) - f_{i,j}^2 \cdot \left((\vec{p}_i\vec{p}_j)-p_{ij}\right)\right), \tag{6}$$

$$p_{ij} = \frac{(\vec{p}_i\vec{r}_{ij})(\vec{p}_j\vec{r}_{ij})}{r_{ij}^2},\ \vec{r}_{ij} = \vec{r}_i - \vec{r}_j,\ f_{i,j}^1 = \frac{4}{(kr_{ij})^2}\left(\frac{\sin(kr_{ij})}{(kr_{ij})} - \cos(kr_{ij})\right),\ f_{i,j}^2 = \frac{4\sin(kr_{ij})}{(kr_{ij})}.$$

When $i=j$ or $r_i=r_j$, expression (6) equals $f_{i,j} = -\frac{8}{3}\pi(\vec{p}_i\vec{p}_j)$.

According to (5), power radiated by the body is

$$P_{sc} = -\frac{k^4}{32\pi^2\varepsilon\sqrt{\varepsilon\mu}} \mathrm{Re}\left[\sum_{i=1}^{N}\sum_{j=1}^{N} f_{i,j} e^{i(\varphi_i-\varphi_j)}\right] \tag{7}$$

and the power radiated by the source itself is

$$P_{inc} = -\frac{k^4}{12\pi\varepsilon\sqrt{\varepsilon\mu}}|\vec{p}_0|^2. \tag{8}$$

If the body is placed in electromagnetic field, energy of interaction between the body and the field is formed. This energy has the following form:

$$\dot{P}_{int} = -\left(\vec{j}_0\vec{E}^*(\vec{r}_0)\right), \tag{9}$$

where $\vec{j}_0 = -\frac{ik\vec{p}_0}{\sqrt{\varepsilon\mu}}$ is the density of current in the source and $\vec{E}^*(\vec{r}_0)$ is a complex conjugate to the field formed by the body in source point $\vec{r}_0$.

The average of this quantity during a period is

$$P_{int} = -\frac{1}{2}\mathrm{Re}\left[\vec{j}_0\vec{E}^*(\vec{r}_0)\right]. \tag{10}$$

So, to find total radiated power we need to add power of free source to the interaction power between the body and the source:

$$P_{source} = P_{inc} + P_{int}. \quad (11)$$

In case of lossy dielectric media, absorbed energy can be derived from the energy balance equation:

$$P_{abs} = P_{source} - P_{total}. \quad (12)$$

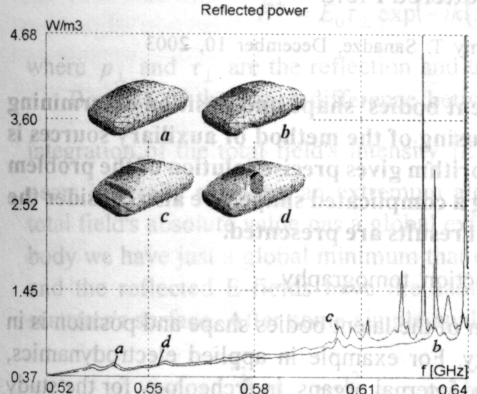

Fig.1. Dependency of energy on frequency for several simplified car models.

Comparison of analytical solution (5) to numerical integration method for diffraction problem on sphere-shaped metallic body has been performed using the M.A.S. Number of dipoles was 588. Time required to obtain analytical solution was 0.34sec as compared to 1.5hrs required for the numerical solution on Pentium III 800 MHz PC.

The obtained analytical formulas were used in many problems in the area of electromagnetic compatibility, namely, for diffraction problem of a simplified car model [3]. In this problem, dependence of reflected field energy on frequency was studied. Simplified car models (a,b,c,d in Fig.1) were studied. Also, using numerical experiments, frequency characteristic curves have been obtained (Fig. 1). The above mentioned analytical formulas reduced the calculation time significantly, made numerical experiments simple and made the study of system's energy characteristics possible in the real time.

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ფიზიკა

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სამგანზომილებიან დიფრაქციის ამოცანებში სხეულის მიერ გაბნეული ველის ენერგიის განსაზღვრა

რეზიუმე. გამოყენებითი ელექტროდინამიკის თანამედროვე პრაქტიკული ამოცანების ამოსახსნელად დიდი მნიშვნელობა აქვს რიცხვითი ექსპერიმენტების ჩატარებას. ერთი მხრივ, აქტუალურია რიცხვითი მეთოდების ოპტიმიზაცია ამოცანის რეალურ დროში ამოსახსნელად, მეორე მხრივ, საჭიროა ამოცანის ამოხსნის შემდეგ შეფასდეს სისტემის ენერგეტიკული მახასიათებლები და შესწავლილ იქნეს მათი ყოფაქცევა. სისტემის ენერგეტიკული მახასიათებლების ცოდნა აგრეთვე საშუალებას იძლევა შემოწმდეს ამოცანის ამოხსნის სიზუსტე და მართებულობა ენერგეტიკული ბალანსის თვალსაზრისით. ამ მიზნით წინამდებარე სტატიაში წარმოდგენილია სამგანზომილებიან დიფრაქციის ამოცანებში ენერგეტიკული მახასიათებლების გათვლის ეფექტური ანალიზური ფორმულები.