Modified Heat Equation for Thermal Calculation on a Realistic Model

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Abstract - Primary intent of this study is development of realistic vascular structure and simulation of the blood perfusion in capillary. It is needed for precise thermal analysis using the modified bio-heat equation to provide better prediction of thermal response of tissues exposed to RF energy then in conventional Pennes model [1].

Particularly, motivation of this paper is to use previously presented modified bio-heat equation [2-4] for heat exchange and temperature rise simulation on a realistic human head model.

Introduction.
Since highly detailed models of real vessel networks in human tissue are not still obtained, there is a need for computer algorithm that can build artificial vasculature. Efforts to generate artificial vascular networks have been motivated by interest in angiogenesis and study of hypothermia.

This paper is continuation of series earlier works where a new algorithm of thermal processes modelling in human tissue was introduced [2-4]. The traditional bio-heat equation (1) [1] was modified in order to account for heat transfer by blood flow.

\[
c_p \rho \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \rho \text{SAR} + A - B(T - T_b) \quad (1)
\]

In modified equation (2), \(T_b\) is not constant anymore, and is changed according to heat transfer by capillary blood flow [2-4].

\[
c_p \rho \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \rho \text{SAR} + A(r, t) - \alpha\left(T - T_b(r, t)\right) \quad (2)
\]

According to which blood could be heated or cooled in the tissue, becoming a convection agent.

Vascular Network.
Construction of vascular system can be split into two subtasks: generation of relatively large vessels [2] and generation of vector blood flow for each point of observed area [3], [4]. Generation of vasculature is performed from root points (points where arteries and veins are allowed to enter the tissue area from outside) moving smoothly in vector field. This vector field is created by points of already constructed vessels and boundary of the tissue domain. Suggested methodology significantly differs from DIVA [5] algorithm where construction of vasculature begins from the endpoints.
Blood velocity in capillary.
Having the discrete vascular network, the next step is to compute the blood velocity vector’s field with assumption that high arterial pressure and low venous pressure determine blood capillary perfusion in each point of tissue. In order to construct continuous blood flow vector field two methods can be considered: the MAS and FDTD method [4]. The MAS is very fast compared to FDTD method but works only for simple, smooth form surface geometries. FDTD can be used for a grid with a virtually any shape.

Validation on a Simplified model.
Obtained results for elementary case are presented on Fig.1. As it can be seen the maximal temperature within the area is lower when blood perfusion it is taken into account and its location is shifted along the blood velocity flow Fig.1 b). It should be noted that modified equation (2) works only if separate parts or organs of human body are considered. In order to simulate whole body we must omit the last condition in (2). Blood temperature $T_b$ would not be constant at arterial endpoints and blood would act completely like an agent (which only transfers heat). But if we consider an organ or a relatively small part of the body, whose surface is significantly smaller than the whole body’s surface, we may say, that blood enters into the studied volume with constant temperature.

![Fig.1. Steady states for a model: (a) without and (b) with blood perfusion.](image)

Considering all stated above we will use proposed approach to calculate temperature and temperature rise on a more realistic model.

Temperature and temperature rise simulations on a realistic model.
Following case was considered: a model of human head [8] exposed to EM field at 450 MHz, radiated by a dipole antenna with length $\lambda/4$. There are two possible ways to calculate temperature rise on this model [7]. We will calculate temperature rise according to penne’s bio-heat equation and according to modified equation (2).

Interaction of vessels with tissue can be calculated in various ways: calculating heat exchange between blood and tissue through vessel boundary or assigning vessel material with high perfusion rate to corresponding cells in the model. Since the velocity of blood flow in large vessels is relatively high, first approach implies strict...
requirements on time step. The second approach does not change the calculation time step, since vessel materials are already present in the model.

Fig. 2. a). Original model, and b). model with generated vessels.

Fig. 3. Steady state for: a) Pennes model and b) for modified equation. Since the difference between temperature distributions is really small these models are in good agreement.

Thus in order to calculate temperature distribution and temperature rise due to EM exposure first we change the model’s grid according to generated vascular network. Fig. 2. Illustrates how the model is changed, the white dots are vessels, appearing as dots since only one slice is being shown. When steady state is reached in both cases we can observe that maximal temperature within the volume is slightly (0.04°C) smaller when blood perfusion it taken into
account. Blood’s flow through the capillary cools the boundary surface resulting in lower temperature at the boundary. Since the blood flow cools down the boundary and redistributes heat within the volume, the location of maximal temperature may change. In the studied model for conventional Penne’s bio-heat equation maximal temperature is observed in the mouth [7], according to modified equation it is in the brain. The numerical estimation of mentioned temperature differences between models is about 0.1°C, thus the new model retains temperature within acceptable range.

![Fig. 4. Temperature rise for: a) Pennes model and b) modified model. SAR normalized to 1W input power](image)

On Fig. 4. it can be seen how blood perfusion affects temperature rise. With blood flow considered temperature rise is smaller (0.5°C in studied case) and its distribution within the volume is changed. It should be noticed that that maximal temperature rise in modified model may be shifted compared to its position in the original model. The difference between temperature rises Δ, depends on particular case, it may rise with rise of SAR or decrease if the point is deeper inside of the model.

An interesting effect can be observed if SAR is augmented. With in increase of SAR, the difference between temperature rise Δ, calculated using Penne’s equation and modified equation (2) increases. At Fig. 3, where the SAR was normalized to input power of 1W, the Δ is 0.83 °C, and if SAR is normalized to input power of 2W Fig. 4, the Δ significantly increases and becomes 1.57°C (Fig 4.).

In studied model maximal temperature rise was inside of the model with discretization of 1mm. In order to use modified equation (2) to calculate temperature rise in thin layers, the discretization should be small enough.

Conclusions.
The equation (2) solved in conjunction with the discrete vascular model lead to fast numerical algorithms for predicting heat exchange in large scale problems. This approach however requires the knowledge of blood velocity distribution throughout
the tissue as well as geometrical model of vascular structure. Availability of such realistic models is a limiting factor in solving practical problems. The novelty of the proposed method consists in algorithm for vascular structure construction and accounting for capillary blood flow in heat transfer in the tissue via convection. It could be mentioned also, that application of the equation (2) satisfies the heat energy conservation automatically.

Calculations show that in the steady state maximal temperatures compared to conventional Pennes equation are relatively the same. But temperature and temperature rise during EM exposure, calculated according to modified model, are lower. It is also shown, that with the increase of SAR the according to the modified model temperature rise is even lower.

Since introduced algorithm of modelling relies on randomly generated vascular network the results may change from model to model and averaging should be considered.

Future work considers further development of the algorithm and subsequent integration of new thermal solver into existing FDTDLab software package.

References.